Recall that if $y = f(x)$, then notationally speaking, $y' = f'(x) = \frac{dy}{dx} = \frac{df}{dx}$.

Basic Rules: If f and g are differentiable functions and c is any constant, then

$$(cf)' = c \cdot f' \qquad (f+g)' = f' + g' \qquad (f-g)' = f' - g'.$$

Power Rule: If $n \in \mathbb{R}$ and $y = x^n$, then $y' = nx^{n-1}$

From here on out, when I refer to a function f, g, etc., it is assumed that these functions are differentiable.

Product Rule: $(fg)' = f'g + fg'$

Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Chain Rule: If $y = f \circ g$, then $y' = f'(g(x))g'(x)$ OR $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ if $y = f(u)$ and $g = u$

Trigonometric Functions:

$$(\sin x)' = \cos x \qquad (\cos x)' = -\sin x$$
$$(\tan x)' = \sec^2 x \qquad (\cot x)' = -\csc^2 x$$
$$(\sec x)' = \sec x \tan x \qquad (\csc x)' = -\csc x \cot x$$

Exponential Functions: $\frac{d}{dx} e^x = e^x \qquad \frac{d}{dx} a^x = a^x \ln a$

Logarithmic Functions: $\frac{d}{dx} \ln x = \frac{1}{x} \qquad \frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

Inverse Trig Functions:

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2} \qquad \frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$$
$$\frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx} \csc^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$$

Chain Rule & Power Rule: $$\frac{d}{dx}(f(x))^n = n(f(x))^{n-1}f'(x)$$

Chain Rule & Trig Functions:

$$(\sin f(x))' = (\cos f(x))f'(x) \qquad (\cos f(x))' = -(\sin f(x))f'(x)$$
$$(\tan f(x))' = (\sec^2 f(x))f'(x) \qquad (\cot f(x))' = -(\csc^2 f(x))f'(x)$$
$$(\sec f(x))' = (\sec f(x)\tan f(x))f'(x) \qquad (\csc f(x))' = -(\csc f(x)\cot f(x))f'(x)$$

Chain Rule & Exponential Functions:

$$\frac{d}{dx}e^{f(x)} = e^{f(x)}f'(x) \qquad \frac{d}{dx}a^{f(x)} = a^{f(x)}f'(x)\ln a$$

Chain Rule & Logarithmic Functions:

$$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)} \qquad \frac{d}{dx}\log_a f(x) = \frac{f'(x)}{f(x)\ln a}$$

Chain Rule & Inverse Trig Functions:

$$\frac{d}{dx}\sin^{-1} f(x) = \frac{f'(x)}{\sqrt{1-(f(x))^2}} \qquad \frac{d}{dx}\cos^{-1} f(x) = \frac{-f'(x)}{\sqrt{1-(f(x))^2}}$$
$$\frac{d}{dx}\tan^{-1} f(x) = \frac{f'(x)}{1+(f(x))^2} \qquad \frac{d}{dx}\cot^{-1} f(x) = \frac{-f'(x)}{1+(f(x))^2}$$
$$\frac{d}{dx}\sec^{-1} f(x) = \frac{f'(x)}{f(x)\sqrt{(f(x))^2-1}} \qquad \frac{d}{dx}\csc^{-1} f(x) = \frac{-f'(x)}{f(x)\sqrt{(f(x))^2-1}}$$

4 Cases for Exponents & Bases (a, b are constants)

1. $\frac{d}{dx}a^b = 0$
2. $\frac{d}{dx}(f(x))^b = b(f(x))^{b-1}f'(x)$
3. $\frac{d}{dx}a^{g(x)} = a^{g(x)}\ln a \cdot g'(x)$
4. $\frac{d}{dx}(f(x))^{g(x)}$ use logarithmic differentiation

Examples:

1. $\frac{d}{dx}2^3$
2. $\frac{d}{dx}(2x^2-5x+1)^3$
3. $\frac{d}{dx}(2^{\sin x})$
4. $\frac{d}{dx}(2x^2-5x+1)^{\sin x}$