Inverse Trigonometry

\[ \arcsin x = \sin^{-1} x = y \iff \sin y = x, \ y \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \]

\[ \sin y = \frac{\text{opp}}{\text{hypo}} = \frac{x}{1} \implies \cos y = \sqrt{1 - x^2} \]

OR \[ 1 - x^2 = 1 - \sin^2 y = \cos^2 y \implies \cos y = \sqrt{1 - x^2} \]

only the pos. square root since angles in QI or QIV

\[ \implies x = a \sin \theta \quad \sin \theta = \frac{x}{a} \quad \theta \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \implies \cos \theta \geq 0 \]

\[ \arctan x = \tan^{-1} x = y \iff \tan y = x, \quad y \in \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \]

\[ \implies x = a \tan \theta, \quad \tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{x}{a}, \quad \theta \in \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \implies \sec \theta > 0 \]

\[ \arccos x = \cos^{-1} x = y \iff \cos y = x, \quad y \in \left[ 0, \frac{\pi}{2} \right] \text{ or } \left[ \pi, \frac{3\pi}{2} \right) \]

\[ \implies x = a \sec \theta, \quad \sec \theta = \frac{\text{hypo}}{\text{adj}} = \frac{x}{a}, \quad \theta \in \left[ 0, \frac{\pi}{2} \right] \text{ or } \left[ \pi, \frac{3\pi}{2} \right) \implies \tan \theta \geq 0 \]
Issues with inverse tangent for Polar, Cylindrical and Spherical Coordinates

In Polar Coordinates (and thus in Cylindrical and Spherical Coordinates), we have the formula

$$\tan \theta = \frac{y}{x}.$$  

The angle $\theta$ needs to be between 0 and $2\pi$ radians, but $\tan^{-1} \frac{y}{x}$ gives us the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Thus just using $\tan^{-1} \frac{y}{x}$ only works if the angle is in QI of the $xy$-plane, i.e., if $x$ and $y$ are both positive. We need to adjust if our angle is in QII, QIII or QIV.

\[\theta \in \text{QI: } (x > 0, y > 0)\]
\[\quad \theta = \tan^{-1} \left( \frac{y}{x} \right)\]

\[\theta \in \text{QII: } (x < 0, y > 0)\]
\[\quad \theta = \tan^{-1} \left( \frac{y}{x} \right) + \pi\]

\[\theta \in \text{QIII: } (x < 0, y < 0)\]
\[\quad \theta = \tan^{-1} \left( \frac{y}{x} \right) + \pi\]

\[\theta \in \text{QIV: } (x > 0, y < 0)\]
\[\quad \theta = \tan^{-1} \left( \frac{y}{x} \right) + 2\pi\]

\textbf{Note:} For spherical coordinates, $\phi \in [0, \pi]$ and if we use the formula $z = \rho \cos \phi$, $\cos^{-1} \left( \frac{z}{\rho} \right)$ will give us the correct angle. Thus there aren’t any issues with $\phi$ as there are with $\theta$. 