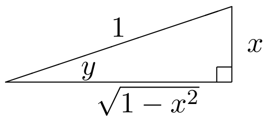
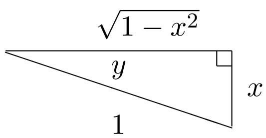
$$\arcsin x = \sin^{-1} x = y \iff \sin y = x,\ y \in \left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]$$



OR



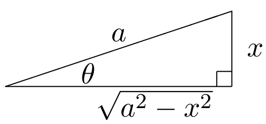
$$\sin y = \frac{\text{opp}}{\text{hypo}} = \frac{x}{1} \Longrightarrow \cos y = \sqrt{1-x^2}$$

$$\text{OR } 1 - x^2 = 1 - \sin^2 y = \cos^2 y$$

$$\Longrightarrow \cos y = \sqrt{1-x^2}$$

only the pos. square root since angles in QI or QIV

$$\Longrightarrow x = a\sin\theta \qquad \sin\theta = \frac{x}{a} \qquad \theta \in \left[-\frac{\pi}{2},\ \frac{\pi}{2}\right] \Longrightarrow \cos\theta \geq 0$$



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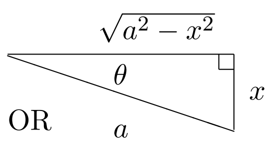


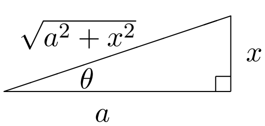
$$x = a\sin\theta$$

$$\Longrightarrow dx = a\cos\theta\, d\theta,$$

$$\sqrt{a^2-x^2} = a\cos\theta$$

$$\arctan x = \tan^{-1} x = y \iff \tan y = x, \qquad y \in \left(-\frac{\pi}{2},\ \frac{\pi}{2}\right)$$

$$\Longrightarrow x = a\tan\theta, \quad \tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{x}{a}, \qquad \theta \in \left(-\frac{\pi}{2},\ \frac{\pi}{2}\right) \Longrightarrow \sec\theta > 0$$



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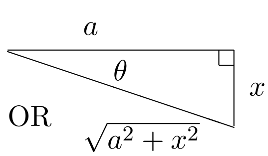


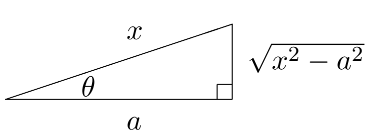
$$x = a\tan\theta$$

$$\Longrightarrow dx = a\sec^2\theta\, d\theta,$$

$$\sqrt{a^2+x^2} = a\sec\theta$$

$$\operatorname{arcsec} x = \sec^{-1} x = y \iff \sec y = x, \qquad y \in \left[0,\ \frac{\pi}{2}\right) \text{ or } \left[\pi,\ \frac{3\pi}{2}\right)$$

$$\Longrightarrow x = a\sec\theta, \quad \sec\theta = \frac{\text{hypo}}{\text{adj}} = \frac{x}{a}, \qquad \theta \in \left[0,\ \frac{\pi}{2}\right) \text{ or } \left[\pi,\ \frac{3\pi}{2}\right) \Longrightarrow \tan\theta \geq 0$$



$$x = a\sec\theta$$

$$\Longrightarrow dx = a\sec\theta\tan\theta\, d\theta,$$

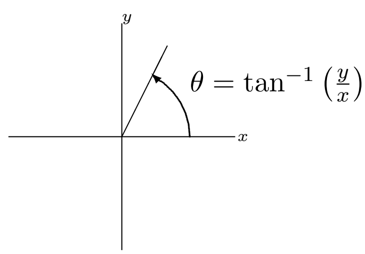
$$\sqrt{x^2-a^2} = a\tan\theta$$

Issues with inverse tangent for Polar, Cylindrical and Spherical Coordinates
In Polar Coordinates (and thus in Cylindrical and Spherical Coordinates), we have the formula

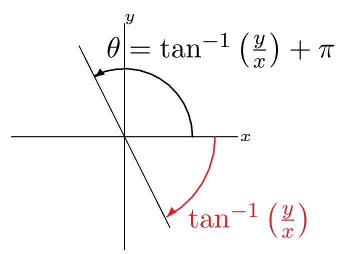
$$\tan\theta = \frac{y}{x}.$$

The angle θ needs to be between 0 and 2π radians, but $\tan^{-1}\frac{y}{x}$ gives us the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Thus just using $\tan^{-1}\frac{y}{x}$ only works if the angle is in QI of the xy-plane, i.e., if x and y are both positive. We need to adjust if our angle is in QII, QIII or QIV.

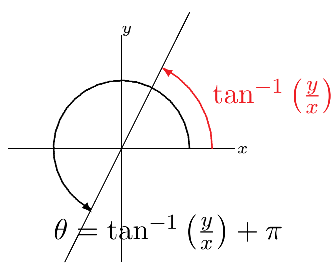
$\theta \in$ QI: $(x > 0,\ y > 0)$



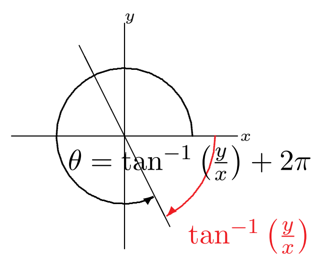
$\theta \in$ QII: $(x < 0,\ y > 0)$



$\theta \in$ QIII: $(x < 0,\ y < 0)$



$\theta \in$ QIV: $(x > 0,\ y < 0)$



Note: For spherical coordinates, $\phi \in [0, \pi]$ and if we use the formula $z = \rho\cos\phi$, $\cos^{-1}\left(\frac{z}{\rho}\right)$ will give us the correct angle. Thus there aren't any issues with ϕ as there are with θ.