Office Hours for Week of April 27-May 1

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<td>as usual</td>
<td>2-3:30</td>
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<td>10:30-11:30, 2-3</td>
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The final for section 1 is on Wednesday, April 29 at 9AM. The final for section 2 is on Friday, May 1 at 1PM. **BOTH FINALS ARE IN OUR MWF CLASSROOM, 007 KH.** You may take the final with the other section, but you MUST email me by 5:00 on Tuesday, April 28. There are limited seats, so this is on a first-come, first-serve basis.

Roughly 60% of the exam will be from Chapters 4 and 5, the other 40% from Chapters 2 and 3.

The same formulas for derivatives given for Exam 2 will be given to you on this exam. I expect you to know how to compute areas of triangles, rectangles and circles and the volume of a rectangular box. Any other area or volume formula will be given to you as needed. I also expect you to know basic antiderivatives and indefinite integrals (see table on page 392).

Types of problems to expect:

- Given certain graphs, answer some questions about limits and continuity.
- Computing derivatives.
  - Some will have no or little partial credit, others will have partial credit. **At least one will involve using the definition of the derivative.**
- Computing limits (2 or 3)
- Applications/interpretations of derivatives.
  - position/height, velocity and acceleration
  - Equations of tangent lines
  - Absolute max/mins
  - Curve sketching (intervals of increase/decrease, concavity, max/mins, etc.)
- Antiderivative question(s) (remember the $+C$!)
- Word problems (may have more than one of each!)
  - related rate
  - optimization
  - business applications
- Using Riemann sums to estimate values of integrals
- Derivatives of integrals (using FTC)
- Evaluating integrals (definite or indefinite) by using:
  - Properties of integrals and given information
  - Interpreting integrals as areas
  - FTC (remember the $+C$ for indefinite integrals!)
  - Substitution rule

What follows are some more practice problems
1. Evaluate the following limits.

(a) \( \lim_{x \to \infty} \frac{3x^2}{1 + 2x - 5x^2} \)
(b) \( \lim_{x \to 3^+} \frac{2x}{9 - x^2} \)
(c) \( \lim_{x \to 0} \frac{\sin 3x}{\tan \pi x} \)
(d) \( \lim_{x \to \infty} \frac{\ln(2 + x)}{4x} \)

2. Find the critical numbers of \( f(x) = 4x^3 - 9x^2 + 12x + 3 \)

3. Find all local and absolute extrema of \( f(x) = x^4 - 2x^2 + 2 \) on the interval \([-2, 2]\).

4. For what values of the constants \( a \) and \( b \) is the point \((1, 6)\) a point of inflection for the curve \( y = x^3 + ax^2 + bx + 1 \)?

5. Consider the curve \( y = x^4 + \frac{8}{3}x^3 + 2x^2 \). Find all of the intercepts, intervals on which the curve in increasing and decreasing; concave up, concave down; all local extrema and points of inflection.

6. Consider the following information about a function \( y = f(x) \) to answer the following questions.

- Domain of \( f \): \( \{x \in \mathbb{R} \mid x \neq 1\} \)
- \( \lim_{x \to -1^-} f(x) = \infty \), \( \lim_{x \to -1^+} f(x) = -\infty \)
- \( y \)-intercept: \((0, -1)\) \( x \)-intercepts: \((-1, 0), (3, 0)\)
- \( f(2) = -2, f(4) = 1 \)
- \( f'(x) \) is defined for all \( x \neq 1 \) and \( f'(x) = 0 \) when \( x = 0, 2, 4 \)
  - \( f'(x) > 0 \) on \((0, 1) \) and \((1, 4)\)
  - \( f'(x) < 0 \) on \((-\infty, 0) \) and \((4, \infty)\)
- \( f''(x) > 0 \) on \((-\infty, 1) \) and \((2, 3)\)
- \( f''(x) < 0 \) on \((1, 2) \) and \((3, \infty)\)

(a) What are the critical numbers of \( f \)?
(b) On what interval(s) is the graph of \( f \) increasing and decreasing (please specify)?
(c) Where are the local extrema? State the \( x \) value and whether it is a local maximum or minimum.
(d) On what interval(s) is the graph of \( f \) concave up?
(e) State the inflection point(s), if any.
(f) Use this information to sketch the graph of \( f \).

7. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm\(^2\)/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm\(^2\)?

8. Gravel is being dumped from a conveyor belt at a rate of 30 ft\(^3\)/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high? Recall that the volume of a cone is given by \( V = \frac{1}{3} \pi r^2 h \), where \( r \) is the radius of the base and \( h \) is the height.
9. Find two positive numbers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

10. A rectangular storage container with an open top is to have a volume of 10 m$^3$. The length of its base is twice the width. Material for the base costs $10 per square meter. Material for the sides costs $6 per square meter. Find the cost of materials for the cheapest such container.

11. A piece of wire 20 inches long is going to be cut into 2 pieces. One of them will be bent into a square and the other into a circle. Find the lengths of the two pieces of wire so that the sum of the areas is as small as possible. What are the lengths of the two pieces so that the sum of the areas is as large as possible?

12. Let $C(x) = 94500 + 140x$ and $p(x) = -\frac{1}{10}x + 555$ be cost and demand functions for selling a new version of iPods, respectively where $x$ is in the thousands ($x = 1$ means 1000 iPods) and $p(x)$ and $C(x)$ is in dollars.

(a) What price should be charged to maximize revenue? How many iPods will be sold at this price?

(b) What price should be charged to maximize profit? How many iPods will be sold at this price?

13. Find the critical numbers of

(a) $f(x) = |4x - 5|$  

(b) $g(x) = \sqrt[3]{x^2 - x}$

14. Find the absolute minimum and maximum values of $f(x) = \frac{2}{3}x^3 - 3x + 1$ on the interval $[-2, 3]$.

15. Let $f(x) = 3x^2 + x$.

(a) Approximate the area under the curve of $f$ on the interval $[0, 2]$ by using right Riemann sums with $n = 4$.

(b) Use the Fundamental Theorem of Calculus to calculate $\int_{0}^{2} f(x) \, dx$.

16. You throw your graphing calculator across the room, and its horizontal velocity at time $t$ is given by $v(t) = -\frac{1}{5}t^2 + 20$, where $t$ is measured in seconds, and $v(t)$ is in feet per second. Assuming it travels in a straight path, find the displacement of the calculator from $t = 0$ to $t = 2$ seconds.

17. The acceleration of a certain particle moving on a straight path is given by $a(t) = t^4 - \sqrt{t}$, where $t$ is in seconds and $a(t)$ is in meters per second per second. It is known that $v(0) = 2$ m/s. Find an equation for $v(t)$.

18. Let $F(t) = \frac{1}{2}\sin^2 t$. Then $F'(t) = \sin t \cos t$. Use the FTC to find $\int_{\frac{\pi}{2}}^{\pi} \sin t \cos t \, dt$.

19. $\frac{d}{dx} \int_{7}^{x} \sqrt{t^8 + 3} \, dt =$
20. \( \frac{d}{dx} \int_1^{e^{x+7}x-2} \frac{\sin(t+1)}{t^6} \, dt = \)

21. Suppose \( f \) and \( g \) are continuous functions and that \( \int_0^2 f(x) \, dx = \sqrt{2}, \int_0^5 f(x) \, dx = \sqrt{5}, \) and \( \int_0^2 g(x) \, dx = 1. \) Find the following:

(a) \( \int_0^2 (7f(x) - 11g(x)) \, dx. \)

(b) \( \int_2^5 f(x) \, dx. \)

22. The following graph shows the function \( f. \) Evaluate the integrals.

23. \( \int \sqrt{x} - x^{-2} + x^3 + 2x + 5 \, dx = \)

24. \( \int e^x + x^e \, dx = \)

25. \( \int \frac{1}{\theta} \, d\theta = \)

26. \( \int (3 \sin x - 5 \cos x) \, dx = \)

27. \( \int x(x^2 - 3)^{49} \, dx = \)

28. \( \int \frac{e^x}{3e^x - 7} \, dx = \)

29. \( \int_0^{\pi/4} (\sec^2 x) e^{\tan x} \, dx = \)

30. \( \int_{\sqrt{\pi}}^{\sqrt{\pi}/2} t \sin(t^2) \, dt = \)

31. \( \int_1^4 \frac{1}{(3x + 1)^6} \, dx = \)