1. Evaluate the following limits.

(a) \( \lim_{x \to \infty} \frac{3x^2}{1 + 2x - 5x^2} \)

\[
\lim_{x \to \infty} \frac{3x^2}{1 + 2x - 5x^2} = \lim_{x \to \infty} \frac{3}{\frac{1}{x^2} + \frac{2}{x} - 5} = \frac{3}{-5}
\]

OR \( \lim_{x \to \infty} \frac{3x^2}{1 + 2x - 5x^2} \) type \( \infty \)

\[
\begin{align*}
LH & = \lim_{x \to \infty} \frac{6x}{2 - 10x} \text{ type } \infty \\
LH & = \lim_{x \to \infty} \frac{6}{-10} = \frac{-3}{5}
\end{align*}
\]

(b) \( \lim_{x \to 3^+} \frac{2x}{9 - x^2} \)

\[
\lim_{x \to 3^+} \frac{2x}{9 - x^2} = \lim_{x \to 3^+} \frac{2x}{(3 + x)(3 - x)} \\
\approx \frac{6}{6 \cdot (-\text{tiny})} = -\infty
\]

(c) \( \lim_{x \to 0} \frac{\sin 3x}{\tan \pi x} \)

\[
\lim_{x \to 0} \frac{\sin 3x}{\tan \pi x} \text{ type } \frac{0}{0} \\
\begin{align*}
LH & = \lim_{x \to 0} \frac{3 \cos 3x}{\pi \sec^2(\pi x)} \\
& = \frac{3}{\pi}
\end{align*}
\]

(d) \( \lim_{x \to \infty} \frac{\ln(2 + x)}{4x} \)

\[
\lim_{x \to \infty} \frac{\ln(2 + x)}{4x} \text{ type } \frac{\infty}{\infty} \\
\begin{align*}
LH & = \lim_{x \to \infty} \frac{\frac{1}{2 + x}}{4} = \frac{1}{8 + 4x} \\
& = \frac{1}{4}
\end{align*}
\]
2. Find the critical numbers of \( f(x) = 4x^3 - 9x^2 + 12x + 3 \)
\[ f'(x) = 12x^2 - 18x + 12 = 6(2x^2 - 3x + 2) \] is always defined so the only critical numbers are when \( f'(x) = 0 \). Use the quadratic formula: 
\[ x = \frac{3 \pm \sqrt{9 - 4(2)(2)}}{4} = \frac{3 \pm \sqrt{-7}}{4} \]
is undefined for real numbers.

There are none.

3. Find all local and absolute extrema of \( f(x) = x^4 - 2x^2 + 2 \) on the interval \([-2, 2]\).

First, find the critical numbers: 
\[ f'(x) = 4x^3 - 4x = 4x(x^2 - 1) \]
\[ 4x(x^2 - 1) = 4x(x + 1)(x - 1) = 0 \] when \( x = 0, \pm 1 \).

To find local extrema: use either the first or second derivative test:

First derivative test

| \( (-\infty, -1) \) | \( - \) | \( - \) | \( - \) |
| \( (-1, 0) \) | \( - \) | \( + \) | \( - \) | \( + \) |
| \( (0, 1) \) | \( + \) | \( + \) | \( - \) | \( - \) |
| \( (1, \infty) \) | \( + \) | \( + \) | \( + \) | \( + \) |

Second derivative test: \( f''(x) = 12x^2 - 4 \)
\[ f''(-1) = 8 \Rightarrow f \text{ concave up at } x = -1 \Rightarrow \text{local min} \]
\[ f''(0) = -4 \Rightarrow f \text{ concave down at } x = 0 \Rightarrow \text{local max} \]
\[ f''(1) = 8 \Rightarrow f \text{ concave up at } x = 1 \Rightarrow \text{local min} \]

\[ f(-1) = 1 = f(1), \quad f(0) = 2 \]

So local min is 1 at \( x = \pm 1 \) and local max is 2 at \( x = 0 \).

To find the absolute extrema, compare \( f \) evaluated at the endpoints and critical numbers:

\[ f(-2) = 10 \quad f(-1) = 1 \quad f(0) = 2 \quad f(1) = 1 \quad f(2) = 10 \]

SO absolute max is 10 (at \( x = \pm 2 \)) and absolute minimum is 1 (at \( x = \pm 1 \)).

4. For what values of the constants \( a \) and \( b \) is the point \((1, 6)\) a point of inflection for the curve \( y = x^3 + ax^2 + bx + 1 \)?

\[ a = -3, b = 7 \]

First, need \((1, 6)\) to be on the curve, i.e., need \( 6 = 1 + a + b + 1 \), so \( b = 4 - a \).

\[ y' = 3x^2 + 2ax + b \Rightarrow y'' = 6x + 2a. \]

For \((1, 6)\) to be an inflection point, need \( y'' = 0 \) at \( x = 1 \) and \( y'' \) to change sign at \( x = 1 \).

\( 0 = 6 + 2a \Rightarrow a = -3. \) So \( y'' = 6x - 6 \). We also see that \( y'' < 0 \) when \( x < 1 \) and \( y'' > 0 \) when \( x > 1 \) so there is an inflection point at \( x = 1 \).

\( a = -3 \Rightarrow b = 4 - (-3) = 7. \)
5. Consider the curve \( y = x^4 + \frac{8}{3}x^3 + 2x^2 \). Find all of the intercepts, intervals on which the curve is increasing and decreasing; concave up, concave down; all local extrema and points of inflection.

- **Find the \( x \) and \( y \)-intercepts.**
  - \( x \)-intercepts: when \( y = 0 \).
    
    \[
    0 = x^4 + \frac{8}{3}x^3 + 2x^2 = x^2(x^2 + \frac{8}{3}x + 2) \quad \text{So } x = 0 \text{ is an } x \text{-intercept.}
    \]
    
    Use the quadratic formula on \( x^2 + \frac{8}{3}x + 2 = 0 \):
    
    \[
    x = \frac{-\frac{8}{3} \pm \sqrt{\frac{64}{9} - 8}}{2} = \frac{-\frac{8}{3} \pm \sqrt{-8}}{2} \quad \text{so no solution: } x = 0 \text{ is the only one.}
    \]
  - \( y \)-intercept: when \( x = 0 \).

- **Find the intervals on which \( f \) is increasing and decreasing.**
  
  Increasing on \((0, \infty)\) and decreasing on \((-, 0)\).

  \[
  y' = 4x^3 + 8x^2 + 4x = 4x(x^2 + 2x + 1)
  \]

  \[
  0 = 4x(x^2 + 2x + 1) = 4x(x + 1)^2 \implies x = 0, -1 \text{ are critical numbers.}
  \]

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<th>(-\infty, -1)</th>
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  So increasing on \((0, \infty)\) and decreasing on \((-, 0)\).

- **Find the intervals on which \( f \) is concave up and concave down.**
  
  Concave up on \((-\infty, -1) \& (-1/3, \infty)\). Concave down on \((-1, -1/3)\).

  \[
  y'' = 12x^2 + 16x + 4 = 4(3x^2 + 4x + 1) = 4(3x + 1)(x + 1)
  \]

  \[
  y'' = 0 \text{ when } x = -1/3, -1
  \]

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  SO, concave up on \((-\infty, -1) \& (-1/3, \infty)\). Concave down on \((-1, -1/3)\).

- **Are there any local extrema?** Yes. If so, what are they? \(\text{Local minimum at } (0, 0)\).
  Use either the first deriv. test of second deriv. test.

- **Are there any inflection points?** Yes. If so, what are they? \(\text{(-1,1/3) and (-1/3, 11/81).}\)

6. Consider the following information about a function \( y = f(x) \) to answer the following questions.

- **Domain of \( f \):** \( \{ x \in \mathbb{R} | x \neq 1 \} \)
- **\( \lim_{x \to 1^-} f(x) = \infty, \lim_{x \to 1^+} f(x) = -\infty \)**
- **\( y \)-intercept: (0, -1) \quad x\)-intercepts: (-1, 0), (3, 0)**
• \( f(2) = -2, \ f(4) = 1 \)
• \( f'(x) \) is defined for all \( x \neq 1 \) and \( f'(x) = 0 \) when \( x = 0, 2, 4 \)
  - \( f'(x) > 0 \) on \( (0, 1) \) and \( (1, 4) \)
  - \( f'(x) < 0 \) on \( (-\infty, 0) \) and \( (4, \infty) \)
• \(-\ f''(x) > 0 \) on \( (-\infty, 1) \) and \( (2, 3) \)
  - \( f''(x) < 0 \) on \( (1, 2) \) and \( (3, \infty) \)

(a) What are the critical numbers of \( f \)?

\[ x = 0, 2, 4 \quad (x = 1 \text{ is not since not in the domain}) \]

(b) On what interval(s) is the graph of \( f \) increasing and decreasing (please specify)?

 increasing on \( (0, 1), \ (1, 4) \); decreasing on \( (-\infty, 0), \ (4, \infty) \)

(c) Where are the local extrema? State the \( x \) value and whether it is a local maximum or minimum.

 local min at \( x = 0 \), local max at \( x = 4 \)

(d) On what interval(s) is the graph of \( f \) concave up?

 concave up on \( (-\infty, 1), \ (2, 3) \)

(e) State the inflection point(s), if any.

\( (2, -2), \ (3, 0) \)

(f) Use this information to sketch the graph of \( f \).

 graph not shown - be sure to notice the vertical asymptote

7. The altitude of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm\(^2\)/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm\(^2\)?

\[-1.6 \text{ cm/min.}\]

Let \( h = \) altitude (height) of the triangle, \( b = \) base of triangle and \( A = \) area of the triangle.

\[ \frac{dh}{dt} = 1, \quad \frac{dA}{dt} = 2, \quad \frac{db}{dt} = ? \text{ when } h = 10 \text{ and } A = 100 \]

\[ A = \frac{1}{2} bh \]

\[ \frac{dA}{dt} = \frac{1}{2} \left( \frac{db}{dt} \cdot h + b \cdot \frac{dh}{dt} \right) \]

\[ 2 = \frac{1}{2} \left( \frac{db}{dt} \cdot 10 + 20 \cdot 1 \right) \quad \text{since} \quad 100 = \frac{1}{2} b \cdot 10 \Rightarrow b = 20 \]

\[ 4 = 10 \frac{db}{dt} + 20 \]

\[ \frac{8}{5} = \frac{db}{dt} \]
8. Gravel is being dumped from a conveyor belt at a rate of 30 ft³/min and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high? Recall that the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$, where $r$ is the radius of the base and $h$ is the height.

\[
\frac{6}{5\pi} \text{ ft/min}
\]

Let $d =$base diameter, $r =$base radius, $h =$ height and $V =$ volume of the cone. 
\[d = h \Rightarrow 2r = h \Rightarrow r = \frac{h}{2} \quad \frac{dV}{dt} = 30, \quad \frac{dh}{dt} = ? \text{ when } h = 10\]

\[V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h\]

\[V = \frac{1}{12}\pi h^3\]

\[\frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \cdot \frac{dh}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}\]

\[30 = \frac{1}{4}\pi (100) \frac{dh}{dt} \Rightarrow \frac{6}{5\pi} = \frac{dh}{dt}\]

9. Find two positive numbers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

\[500, 125\]

\[x, y > 0, \quad x + 4y = 1000 \Rightarrow x = 1000 - 4y\]

\[P = xy = (1000 - 4y)y = 1000y - 4y^2, \text{ want to maximize } P. \quad P' = 1000 - 8y\]

\[0 = 1000 - 8y \Rightarrow y = 125\]

\[P'' = -8 \text{ so max at } y = 125. \quad x = 1000 - 4(125) = 500.\]

10. A rectangular storage container with an open top is to have a volume of 10 m³. The length of its base is twice the width. Material for the base costs $10 per square meter. Material for the sides costs $6 per square meter. Find the cost of materials for the cheapest such container.

\[\text{about } $163.54\]

\[V = lwh, \quad l = 2w, \quad V = 10 \Rightarrow 10 = 2w^2 h \Rightarrow h = \frac{5}{w^2}\]

\[C = 2(\text{cost of sides with dim } h, l) + 2(\text{cost of sides with dim } h, w) + \text{cost of base}\]

\[C = 2 \cdot 6(hl) + 2 \cdot 6(hw) + 10(lw)\]

\[C = 12 \cdot \frac{5}{w^2} \cdot 2w + 12 \cdot \frac{5}{w^2} \cdot w + 10 \cdot 2w^2 = \frac{180}{w} + 20w^2 \text{ want to minimize } C:\]

\[C' = -180w^{-2} + 40w\]

\[0 = \frac{-180 + 40w^3}{w^2} \Leftrightarrow 0 = -180 + 40w^3\]

\[w^3 = \frac{9}{2} \Rightarrow w = \sqrt[3]{\frac{9}{2}}\]

\[C'' = 360w^{-3} + 40 > 0 \text{ so min at } w = \sqrt[3]{\frac{9}{2}}\]

\[C(w) = \frac{180}{\sqrt[3]{9/2}} + 20 \left(\sqrt[3]{\frac{9}{2}}\right)^2 \approx $163.54\]
11. A piece of wire 20 inches long is going to be cut into 2 pieces. One of them will be bent into a square and the other into a circle. Find the lengths of the two pieces of wire so that the sum of the areas is as small as possible. What are the lengths of the two pieces so that the sum of the areas is as large as possible?

\[ A_{\text{square}} = \left( \frac{x}{4} \right)^2 = \frac{x^2}{16} \]

\[ 20 - x = 2\pi r \implies r = \frac{20 - x}{2\pi} \]

\[ A_{\text{circle}} = \pi r^2 = \pi \left( \frac{20 - x}{2\pi} \right)^2 = \frac{(20 - x)^2}{4\pi} \]

Maximize/minimize \( A = A_{\text{square}} + A_{\text{circle}}, x \in [0, 20] \)

\[ A' = \frac{2x}{16} + \frac{-2(20 - x)}{4\pi} \quad \text{(chain rule on 2nd term)} \]

\[ = \frac{x}{8} + \frac{x - 20}{2\pi} = \frac{\pi x + 4x - 80}{8\pi} \]

\[ 0 = (\pi + 4)x - 80 \implies x = \frac{80}{\pi + 4} \approx 11.2 \]

Check for max/min by plugging in critical numbers and endpoints \((x = 0, 20)\)

\[ A(0) = \frac{20^2}{4\pi} \approx 31.83 \]

\[ A \left( \frac{80}{\pi + 4} \right) \approx 14.0025 \]

\[ A(20) = 25 \]

Minimum area occurs when cut is made at \( x = \frac{80}{\pi + 4} \) so the piece of length \( \frac{80}{\pi + 4} \)

is made into the square and the other piece into the circle. Maximum area occurs when no cut is made and the entire piece is made into a circle.
12. Let \( C(x) = 94500 + 140x \) and \( p(x) = -\frac{1}{10}x + 555 \) be cost and demand functions for selling a new version of iPods, respectively where \( x \) is in the thousands (\( x = 1 \) means 1000 iPods) and \( p(x) \) and \( C(x) \) is in dollars.

(a) What price should be charged to maximize revenue? How many iPods will be sold at this price?

\[
R(x) = xp(x) = -\frac{1}{10}x^2 + 555x
\]

\[
R'(x) = -\frac{1}{5}x + 555 = 0 \implies x = 2775
\]

\[
R''(2775) = -\frac{1}{5} < 0 \implies \text{max at } x = 2775
\]

\[
p(2775) = 277.50
\]

max revenue is when price is $277.50 and 2,775,000 iPods will be sold at this price

(b) What price should be charged to maximize profit? How many iPods will be sold at this price?

max profit when \( R'(x) = C'(x) \) and \( R''(x) < C''(x) \)

\[
-\frac{1}{5}x + 555 = 140 \implies x = 2075
\]

\[
R''(x) = -\frac{1}{5}, \quad C''(x) = 0 \quad \text{so } R''(x) < C''(x)
\]

\[
p(2075) = 347.5
\]

max revenue is when price is $347.50 and 2,075,000 iPods will be sold at this price

13. Find the critical numbers of

(a) \( f(x) = |4x - 5| \)

Critical numbers are when \( f'(x) = 0 \) or doesn’t exist. Since

\[
f(x) = |4x - 5| = \begin{cases} -4x + 5 & x < \frac{5}{4} \\ 4x - 5 & x \geq \frac{5}{4} \end{cases}
\]

we have \( f'(x) = 4 \) when \( x > 5/4 \) and \( f'(x) = -4 \) when \( x < 5/4 \) and \( f'(5/4) \) doesn’t exist. So the only critical number is \( x = 5/4 \).

(b) \( g(x) = \sqrt[3]{x^2 - x} \)

\[
g(x) = (x^2 - x)^{\frac{1}{3}}
\]

\[
g'(x) = \frac{1}{3}(x^2 - x)^{-\frac{2}{3}}(2x - 1) = \frac{2x - 1}{3\sqrt[3]{(x^2 - x)^2}}
\]

\[
g'(x) = 0 \quad \text{when } 2x - 1 = 0 \implies x = \frac{1}{2}
\]

\[ g'(x) \text{ doesn’t exist when } \sqrt[3]{x^2 - x} = 0 \implies x^2 - x = 0 \implies x(x - 1) = 0 \implies x = 0, 1 \]

So the critical numbers are \( x = 0, \frac{1}{2}, 1 \).
14. Find the absolute minimum and maximum values of \( f(x) = \frac{2}{3}x^3 - 3x + 1 \) on the interval \([-2, 3]\).

First, find the critical numbers.
Second, check the values of \( f \) at these critical numbers and at the endpoints of the interval.
Lastly, compare and determine the max and min values.

\[
f'(x) = 2x^2 - 3
\]

\[
2x^2 - 3 = 0 \implies x = \pm \sqrt{\frac{3}{2}}
\]

\[
f(-2) = \frac{5}{3} \approx 1.667
\]

\[
f\left(-\sqrt{\frac{3}{2}}\right) = \frac{2}{3} \left(-\sqrt{\frac{3}{2}}\right)^3 + 3\sqrt{\frac{3}{2}} + 1 = -\frac{2}{3} \cdot \frac{3}{2} \sqrt{\frac{3}{2}} + 3\sqrt{\frac{3}{2}} + 1
\]

\[
= 2\sqrt{\frac{3}{2}} + 1 \approx 3.4495
\]

\[
f\left(\sqrt{\frac{3}{2}}\right) = -2\sqrt{\frac{3}{2}} + 1 \approx -1.4495
\]

\[
f(3) = 10
\]

So the maximum value on \([-2, 3]\) is 10, minimum value is \(-2\sqrt{\frac{3}{2}} + 1\).

15. Let \( f(x) = 3x^2 + x \).

(a) Approximate the area under the curve of \( f \) on the interval \([0, 2]\) by using right Riemann sums with \( n = 4 \).

\[
\Delta x = \frac{2 - 0}{4} = \frac{1}{2} \quad x_1^* = \frac{1}{2}, \; x_2^* = 1, \; x_3^* = \frac{3}{2}, \; x_4^* = 2
\]

\[
R_4 = \sum_{k=1}^{4} f(x_k^*) \Delta x = \sum_{k=1}^{4} f(x_k^*) \frac{1}{2} = \frac{1}{2} \sum_{k=1}^{4} f(x_k^*)
\]

\[
= \frac{1}{2} \left[ f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2) \right]
\]

\[
= \frac{1}{2} \left( \frac{5}{4} + 4 + \frac{33}{4} + 14 \right) = \frac{110}{8} = \frac{55}{4}
\]

(b) Use the Fundamental Theorem of Calculus to calculate \( \int_0^2 f(x) \, dx \).

\(3x^2 + x\) is continuous on \([0, 2]\) with antiderivative \(3\frac{x^3}{3} + \frac{x^2}{2} = x^3 + \frac{1}{2}x^2\) so by the FTC,

\[
\int_0^2 3x^2 + x \, dx = \left. \left( x^3 + \frac{1}{2}x^2 \right) \right|_0^2 = 8 + \frac{1}{2} \cdot 4 - 0 = 10
\]
16. You throw your graphing calculator across the room, and its horizontal velocity at time \( t \) is given by \( v(t) = -\frac{1}{5}t^2 + 20 \), where \( t \) is measured in seconds, and \( v(t) \) is in feet per second. Assuming it travels in a straight path, find the displacement of the calculator from \( t = 0 \) to \( t = 2 \) seconds.

\[
\text{Displacement} = \int_0^2 v(t) \, dt = \int_0^2 \left( -\frac{1}{5}t^2 + 20 \right) \, dt
\]

\[
= \left. \left( -\frac{1}{5} \cdot \frac{t^3}{3} + 20t \right) \right|_0^2 = -\frac{8}{15} + 40 - 0
\]

\[
= \frac{592}{15} \approx 39.47 \text{ ft}
\]

17. The acceleration of a certain particle moving on a straight path is given by \( a(t) = t^4 - \sqrt{t} \), where \( t \) is in seconds and \( a(t) \) is in meters per second per second. It is known that \( v(0) = 2 \text{ m/s} \). Find an equation for \( v(t) \).

\[
v(t) = \int a(t) \, dt
\]

\[
= \int t^4 - t^{\frac{1}{2}} \, dt
\]

\[
= \frac{1}{5}t^5 - \frac{2}{3}t^{\frac{3}{2}} + C
\]

\[
v(0) = 2 = C \Rightarrow v(t) = \frac{1}{5}t^5 - \frac{2}{3}t^{\frac{3}{2}} + 2
\]

18. Let \( F(t) = \frac{1}{2} \sin^2 t \). Then \( F'(t) = \sin t \cos t \). Use the FTC to find \( \int_{\frac{\pi}{2}}^{\pi} \sin t \cos t \, dt \).

Since \( F'(t) = \sin t \cos t \) is continuous on \( [\frac{\pi}{2}, \pi] \), by the FTC we have

\[
\int_{\frac{\pi}{2}}^{\pi} \sin t \cos t \, dt = \int_{\frac{\pi}{2}}^{\pi} F'(t) \, dt
\]

\[
= F(\pi) - F\left(\frac{\pi}{2}\right)
\]

\[
= \frac{1}{2} \sin^2 \pi - \frac{1}{2} \sin^2 \frac{\pi}{2}
\]

\[
= 0 - \frac{1}{2} = -\frac{1}{2}
\]

19. \( \frac{d}{dx} \int_{7}^{x} \sqrt{t^8 + 3} \, dt = \)

Since \( \sqrt{t^8 + 3} \) is continuous on \((-\infty, \infty)\), by the FTCI we have

\[
\frac{d}{dx} \int_{7}^{x} \sqrt{t^8 + 3} \, dt = \sqrt{x^8 + 3}
\]
20. \[ \frac{d}{dx} \int_1^{e^x+7x-2} \frac{\sin(t+1)}{t^6} \, dt = \]

2 ways – both involve the Chain Rule, just using different notation.

**First way:**

Let \( f(x) = \int_1^{e^x+7x-2} \frac{\sin(t+1)}{t^6} \, dt \) and \( g(x) = e^x + 7x - 2 \)

Then \( f(g(x)) = \int_1^{e^x+7x-2} \frac{\sin(t+1)}{t^6} \, dt \) and by the Chain Rule, we have \[ f'(g(x)) = f'(g(x)) g'(x) \]

For \( t \neq 0, \frac{\sin(t+1)}{t^6} \) is continuous so by the FTC we have \[ f'(x) = \frac{\sin(x+1)}{x^6} \]

\[ g'(x) = e^x + 7 \]

\[ f'(g(x)) = \frac{\sin(e^x + 7x - 2 + 1)}{(e^x + 7x - 2)^6} \]

so we have

\[ [f(g(x))]' = \frac{\sin(e^x + 7x - 1)}{(e^x + 7x - 2)^6} \cdot (e^x + 7) \]

**Second way:** Let \( u = e^x + 7x - 2 \). Then \( \frac{du}{dx} = e^x + 7 \)

By the Chain Rule,

\[ \frac{d}{dx} \int_1^{e^x+7x-2} \frac{\sin(t+1)}{t^6} \, dt = \left( \frac{d}{du} \cdot \int_1^u \frac{\sin(t+1)}{t^6} \, dt \right) \left( \frac{du}{dx} \right) \]

\[ = \frac{\sin(u+1)}{u^6} \cdot \frac{du}{dx} \quad \text{by same reason as above using FTC} \]

\[ = \frac{\sin(e^x + 7x - 1)}{(e^x + 7x - 2)^6} \cdot (e^x + 7) \]

21. Suppose \( f \) and \( g \) are continuous functions and that \( \int_0^2 f(x) \, dx = \sqrt{2}, \int_0^5 f(x) \, dx = \sqrt{5}, \) and \( \int_0^2 g(x) \, dx = 1. \) Find the following:

(a) \[ \int_0^2 (7f(x) - 11g(x)) \, dx. \]

\[ = 7 \int_0^2 f(x) \, dx - 11 \int_0^2 g(x) \, dx = 7\sqrt{2} - 11 \cdot 1 = 7\sqrt{2} - 11 \]

(b) \[ \int_2^5 f(x) \, dx. \]

\[ = \int_0^5 f(x) \, dx - \int_0^2 f(x) \, dx = \sqrt{5} - \sqrt{2} \]

22. The following graph shows the function \( f \). Evaluate the integrals.

Graph not shown in solutions; compute in terms of areas.
(a) $\int_0^2 f(x) \, dx = 2$
(b) $\int_2^3 f(x) \, dx = 0$
(c) $\int_0^4 f(x) \, dx = 1$
(d) $\int_0^5 f(x) \, dx = -1$
(e) $\int_3^4 f(x) \, dx = 1$

23. $\int \sqrt[3]{x} - x^{-2} + x^3 + 2x + 5 \, dx =$

\[
\int \sqrt[3]{x} - x^{-2} + x^3 + 2x + 5 \, dx = \int x^{\frac{1}{3}} \, dx - \int x^{-2} \, dx + \int x^3 \, dx + 2 \int x \, dx + \int 5 \, dx
\]
\[
= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} - \frac{x^{-1}}{-1} + \frac{x^4}{4} + 2 \cdot \frac{x^2}{2} + 5x + C
\]
\[
= \frac{3 x^{\frac{4}{3}}}{4} + x^{-1} + \frac{x^4}{4} + x^2 + 5x + C
\]

24. $\int e^x + x^e \, dx = \int e^x \, dx + \int x^e \, dx = e^x + \frac{x^{e+1}}{e+1} + C$

25. $\int \frac{1}{\theta} \, d\theta = \ln |\theta| + C$

26. $\int (3 \sin x - 5 \cos x) \, dx = 3 \int \sin x \, dx - 5 \int \cos x \, dx = -3 \cos x - 5 \sin x + C$

27. $\int x(x^2 - 3)^{49} \, dx =$

Let $u = x^2 - 3 \implies du = 2x \, dx$ OR $\frac{1}{2} \, du = x \, dx$

\[
\int x(x^2 - 3)^{49} \, dx = \int u^{49} \frac{1}{2} \, du = \frac{1}{2} \int u^{49} \, du
\]
\[
= \frac{1}{2} \cdot \frac{u^{50}}{50} + C = \frac{1}{100} (x^2 - 3)^{50} + C
\]

28. $\int \frac{e^x}{3e^x - 7} \, dx =$

Let $u = 3e^x - 7 \implies du = 3e^x \, dx$ OR $\frac{1}{3} \, du = e^x \, dx$

\[
\int \frac{e^x}{3e^x - 7} \, dx = \int \frac{1}{u} \cdot \frac{1}{3} \, du = \frac{1}{3} \int \frac{1}{u} \, du
\]
\[
= \frac{1}{3} \ln |u| + C = \frac{1}{3} \ln |e^x - 7| + C
\]

29. $\int_0^\pi (\sec^2 x) \, e^{\tan x} \, dx =$

Let $u = \tan x \implies du = \sec^2 x \, dx$
When $x = 0$, $u = \tan 0 = 0$. When $x = \pi/4$, $u = 1$.

\[
\int_0^{\pi/4} \sec^2 x e^{\tan x} \, dx = \int_0^1 e^u \, du = e^u \bigg|_0^1 = e - 1
\]

30. \[
\int_{\frac{\sqrt{\pi}}{2}}^{\frac{\pi}{2}} t \sin(t^2) \, dt =
\]
Let $u = t^2 \Rightarrow du = 2t \, dt$ OR $\frac{1}{2} \, du = t \, dt$

When $t = \frac{\sqrt{\pi}}{2}$, $u = \frac{\pi}{4}$. When $t = \sqrt{\pi}$, $u = \pi$

\[
\int_{\frac{\sqrt{\pi}}{2}}^{\frac{\pi}{2}} t \sin(t^2) \, dt = \frac{1}{2} \int_{\frac{\pi}{4}}^{\pi} \sin u \, du = \left[ -\frac{1}{2} \cos u \right]^{\pi}_{\frac{\pi}{4}}
\]
\[
= \left( -\frac{1}{2} \cdot (-1) \right) - \left( -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} \right)
\]
\[
= \frac{1}{2} + \frac{\sqrt{2}}{4}
\]

31. \[
\int_1^4 \frac{1}{(3x+1)^{16}} \, dx =
\]
Let $u = 3x + 1 \Rightarrow du = 3 \, dx$ OR $\frac{1}{3} \, du = dx$ \quad $x = 1 \Rightarrow u = 4$ \quad $x = 4 \Rightarrow u = 13$

\[
\int_1^4 \frac{1}{(3x+1)^{16}} \, dx = \frac{1}{3} \int_4^{13} u^{-16} \, du = \frac{1}{3} \cdot u^{-15} \bigg|_4^{13} = -\frac{1}{45} \cdot \frac{1}{u^{15}} \bigg|_4^{13}
\]
\[
= -\frac{1}{45} \left( \frac{1}{13^{15}} - \frac{1}{4^{15}} \right)
\]