Due: Monday, April 23 at 4 PM

- For this assignment you will create 5 files; `hw8Ober.m`, `rsumOber.m`, and `sruleOber.m`, where "Ober" is replaced by the first four letters of your last name.
- The SCRIPT FILE `hw8Ober.m` should be formatted in such a way to be "publishable" as a webpage. Your webpage should appear on the G-drive in the same folders as your previous assignments. Within a browser, copy and paste the URL for your page into Moodle by the due date/time.
- All other directions are the same as the previous assignments. This homework will also require work on paper to be turned in by the due date and time.

All ERROR CHECKS within your functions should use the `error` command and display a meaningful error message.

1. Create a function `rsumOber.m` that takes as input a function `fstring`, `a`, `b`, `n`, `sumType`, and `color`. The commands `syms`, `sym`, and `double` may be useful or necessary. The `switch` selection statement may also come in handy for this assignment.

   - The input `fstring` will be a string for the function $f(x)$ that we want to integrate numerically. The string is in single quotes using standard MATLAB notation WITHOUT component-wise calculations. No error checking will be done on this.
   - The inputs `a` and `b` are the lower and upper bounds, respectively for the integration. An error check will check that `a` < `b`; if not, an appropriate error message will be displayed.
   - The input `n` will be the number of rectangles/subintervals that will be made for the numerical integration (Riemann Sum). An error check will check that `n` is a positive integer; if not, an appropriate error message will be displayed.
   - The input `sumType` will determine which type of Riemann Sum will be calculated and plotted. It will be a string in single quotes and will be either `'right'`, `'left'`, `'mid'` or `'trap'`; if not, an appropriate error message will be displayed.
   - The input `color` will specify which color will be used to fill in the rectangles. It could be a vector or a string names for a color. No error checking will be done on this input.

   The function will have 1 output; the calculated Riemann Sum (using Right Riemann sum if `sumType` = `'right'`, Left Riemann sum if `sumType` = `'left'`, Midpoint Rule if `sumType` = `'mid'`, and the Trapezoidal Rule if `sumType` = `'trap'`). The value that is the output should NOT be symbolic - thus you may need to use the `double` command. The function will also plot the rule (depending on the value of `sumType`) of $y = f(x)$ from $x = a$ to $x = b$ using n subdivisions. You will use the `fill` command. Both the edges of the rectangles/trapezoids and the function $y = f(x)$ will be plotted in black. There will be no titles, axis labels, or other axis commands within the function; the user can add these outside of the function.

   Have a link on your `hw8Ober` webpage to this function `rsumOber`.

2. For $f(x) = \sin^4(\pi x) + 2x$, figure out the following by hand on paper and/or using MATLAB to help with some calculations.. Any calculations not done by hand should be shown in MATLAB within your `hw8Ober` file ("show your work").

   (a) Approximate $\int_1^3 f(x)\,dx$ using a Left Riemann Sum and $n = 4$.

   (b) Approximate $\int_1^3 f(x)\,dx$ using a Right Riemann Sum and $n = 4$.

   (c) Approximate $\int_1^3 f(x)\,dx$ using a Midpoint Rule and $n = 4$.

   (d) Approximate $\int_1^3 f(x)\,dx$ using a Trapezoidal Rule and $n = 4$.

3. (a) Use the `subplot` command and your function to show all four Riemann sums with $n = 4$ on the same figure using your `rsumOber` function for $\int_1^3 f(x)\,dx$ using $f(x)$ above. This will be a 2×2 figure where the top row will be the Left and Right Riemann Sums and the second row will be the Midpoint Rule and Trapezoidal Rule. Have the Left Riemann Sum be in Loyola Green ($[0, 0.4078, 0.3412]$), the Right Riemann Sum in yellow, the Midpoint Rule in red with `alpha(0.3)`, and the Trapezoidal Rule in gray ($[0.75, 0.75, 0.75]$). Make sure you add titles specifying which rule is which.

   (b) Compare the answers you get from your function with your answers in the previous problem. Are they what you expected?

4. Use the `subplot` command to plot the one of the rules (your choice but use the same rule for all four within these subplots) using your `rsumOber` function on a figure with 2×2 subplots: $n = 4$, $n = 8$, $n = 30$, $n = 75$ for $\int_1^3 f(x)\,dx$.

5. Create a function `sruleOber.m` that takes as input a function `fstring`, `a`, `b`, and `n`.

   - The input `fstring` is a string for the function $f(x)$ in single quotes using standard MATLAB notation WITHOUT component-wise operations, just as in `rsumOber`. No error check is done on this input.
   - The inputs `a` and `b` are the lower and upper bounds, respectively for the integration. An error check will check that `a <b`; if not, an appropriate error message will be displayed.
   - The input `n` will be the number of rectangles/subintervals that will be made for the Simpson's Rule. An error check will check that `n` is a positive EVEN integer; if not, an appropriate error message will be displayed.

   The commands `syms`, `sym`, `sum`, `subs`, and `double` will be useful or necessary. The output of the function will be the approximation of the integral of $f(x)$ from a to b using the Simpson's Rule on the given n. Again, it should be a numerical approximation so you may need to use the `double` command. Have a link on your `hw8Ober` webpage to this function `sruleOber`.

6. Create a function `splotOber.m` that has the same inputs (and error checks) as `sruleOber`. This function will plot the Simpson's Rule of $y = f(x)$ from $x = a$ to $x = b$ using n subdivisions. You will use the `syms`, `subs`, `polyfit`, and the `polyval` commands. The approximating quadratics will be plotted in blue, and the curve $y = f(x)$ will be in black. Have the domains in the plots for the approximating quadratics be 0.05 beyond the x_k used in the approximations.

For example, if the first quadratic is approximating the curve from 1 to 1.5, then have the plot of the quadratic be from 0.95 to 1.55. You will also plot the subdivisions in black; these will be vertical lines from $(x_k, 0)$ to (x_k, y_k) for each of $x_0, x_1, \dots, x_n$.

7. We will investigate Simpson's Rule to estimate $\int_1^3 f(x)\,dx$ for $f(x)$ above with $n = 2$ and $n = 4$. For the subdivisions, figure out what the FIRST approximating quadratics would be. This should be done on paper and turned in, showing all work, explaining it clearly, and using exact values. Any calculations using technology should be done in MATLAB and shown within the `hw8Ober` file. The answers for the approximating quadratics (and the subintervals they are for) should appear in the text of the webpage. Use the interval $[-h, h]$ and figure out the coefficients A, B, and C based on the values of y_k, y_{k+1}, and y_{k+2} from the notes. That quadratic is based on the middle x-value equaling 0. Use a horizontal shift so the the middle value is now at x_{k+1}.
   (a) Figure out the FIRST approximating quadratic for $n = 2$.
   (b) Figure out the FIRST approximating quadratic for $n = 4$.

8. Check your answers in the above problem using the command `polyfit` and even `poly2sym`, clearly labeling your answers in the webpage.
   (a) Check for $n = 2$.
   (b) Check for $n = 4$.

9. Use your `sruleOber` function to approximate $\int_1^3 f(x)\,dx$ with the Simpson's Rule using $n = 2$, $n = 4$, $n = 8$, and $n = 16$. Also, use `subplot` as in previous plots with your `splotOber` to visualize the Simpson's Rule for these four numerical approximations.