Due by the 4:00 PM Wednesday, December 17

Instructions: You must work on this project <u>**entirely**</u> on your own.

The work you hand in should be presentable and easy to read in the form of a report that summarizes your findings rather than a "question/answer" format. Any calculations or formulas used should be explained, and supported by the graphs. Do not highlight each of the tasks, but you should highlight the different sections (the business or government side). Any graphs you include should be appropriately labeled and within the report. Any m-files that you write and/or MATLAB output that shows how your programs work should be included as an Appendix to your report. In writing the report, think of your audience as being your boss at the company, the city council, or a consumer watch-dog group, etc. that does not know of these functions, but with some basic explanation should understand what you are presenting. The appendices are for the more computationally/mathematically savvy person that wants to double-check your report.

Your score will depend on overall format (including graphs and such), writing, accuracy and the efficiency/accuracy of the MATLAB programming involved. All graphs should be as "professional looking" as possible by using MATLAB. Titles should be descriptive, the domains/axes should be adjusted accordingly to see a complete picture of what is going on, complete with legends, labels and such. Make sure that the axes are labeled appropriately (percentage of households? number of households in the 100s?, price in dollars? price in 100s of dollars? etc.) Any mathematical computations should be explained within the report. Basic equations should be given within the report with a basic overview of the process. The actual mathematical work should be cited and appear in an appendix. Thus you'll have 2 main sections of the report: business and government, and 2 or more appendices. At least one of the appendices is the MATLAB code used to generate your plots and/or answers and at least one of the appendices may be your typed up or hand-written mathematical work. Keep in mind that your answers should make sense. For example, if one of the answers mathematically comes to 15.3 people, do you use 15 or 16? Whichever you choose should be explained within the report.

1 Introduction

This project deals with an economics application of monopoly pricing. When there is only one company that offers a certain service or product, then it is called a monopoly. Examples include local telephone services, electric and gas companies and cable providers. Monopolists have the advantage over other companies that must compete, since without regulation by local or federal governments, they can potentially control the price of their service or product by controlling the quantity produced. A product that is in short supply will fetch a higher price if the demand for it is high. Conversely, if the product is easy to come by then its price will be low. Monopolies could be detrimental to consumers if the company was not interested in providing enough of their product or service, but rather they were interested in providing just enough to maximize their profits.

Some companies that have received a lot of attention as potential monopolies are cable TV companies. People have been dissatisfied with having to pay extra fees for special cable channels, DVRs

or HD service, etc. The question of regulating the cable companies has been a matter of some concern. In this project, you will answer questions about how monopolies set prices and whether or not the cable companies should be regulated by the government.

2   Going into Business

Suppose a small town has offered to give you the rights to provide the extended cable TV service to 10,000 households that do not have any cable TV. As a business person you are interested in maximizing your profit, and your cost of providing extended cable TV is $25 per month per household plus $4000 in monthly overhead that is related to maintenance of your equipment (regardless of how many households get your service). Half of the households live in houses and the other half live in apartments. It has been estimated that people living in houses are more likely to order your services, and if you charge a monthly price of p for cable TV then the **percentage** q_h of households living in houses who would pay for cable is given by

$$q_h(p) = \begin{cases} 50 & \text{if } p \le 65 \\ \dfrac{1500 - 10p}{17} & \text{if } 65 < p < 150 \\ 0 & \text{if } p \ge 150. \end{cases}$$

The following expression gives the **percentage** q_a of households living in apartments who will pay for hookup if you charge a price of p:

$$q_a(p) = \begin{cases} \dfrac{120 - p}{2} & \text{if } p < 120 \\ 0 & \text{if } p \ge 120. \end{cases}$$

These are known as *demand functions* or demand curves. Note that the town giving you the franchise will only allow you to set one price for cable TV, regardless of it being in a house or apartment.

Task 1: Create two functions `qh` and `qa` in MATLAB for these demand functions. Each function should take as input either a price p or a vector of prices and return the computed q_h or q_a values for the input. Make a plot of the two demand functions in terms of p on the same axis, for the price range of $0 \le p \le 160$. As a monopolist your goal is to determine what price p you should charge for cable TV so that your profit is maximized. This will be done with a series of tasks described below.

Task 2: Your revenue is the amount of money that you take in from cable TV in one month. It is equal to the price you charge times the number of cable TV hookups you sell. Express the revenue as a function of p, keeping in mind that the **percentage** of sales to houses and apartments is given by q_h and q_a, respectively. Plot the revenue as a function of p for the price range $0 \le p \le 160$.

Task 3: Express your total monthly cost in providing cable service in terms of p. Remember, the costs are $25 per hookup per month plus $4000 in overhead costs. Plot the total cost as a function of p for the price range $0 \le p \le 160$.

Task 4: You are now (potentially) ready to go into business. The monthly profit from running your cable company is your total revenue from sales minus the total cost to you of providing the service. Express your profit as a function of the price p you charge for cable and obtain a plot of it for the price range $0 \leq p \leq 160$.

Task 5: The plot from Task 4 should indicate there is an *optimal* price to maximize your profit. Calculuate *mathematically* what this optimal price is and the maximum profit (your answer should agree with your graph!)

Task 6: At the optimal price you are going to charge for cable, how many households living in houses are paying for your service? How many households living in apartments are paying? Suppose now that the town demands that you pay a monthly fee for this privilege. How much are you willing to pay so that your company can still remain profitable? (There is no one exact answer, since the fee would have to vary from 0 to the maximum profit you will be making – what would be a reasonable, business-oriented answer?)

3 Going into politics

Suppose now that instead of going into business, you are part of the government of the small town that has given someone the right to provide cable TV. This company has been in business for several years and has led to some complaints. The biggest complaint is that for people living in apartments, the price is so high that they cannot afford it. Also, it is cheaper for the cable company to provide and maintain service to an apartment building than to a neighborhood. The company claims that they must charge a high price to remain in business, but suggests that the city government allows the company to charge different prices for apartments and houses.

Task 7: Suppose that the monopolist can charge a price p_a to households living in apartments and a price p_h to households living in houses. What pair of prices maximizes profit? How many houses and apartments are served at these prices? How large of a service fee could the town charge and still allow the monopolist to stay in business?

Note: to answer the questions in Task 7, you will need to maximize a function of two variables. This can be done in a number of ways: multivariable calculus, a MATLAB built-in command, or a graphical approach (which is not as efficient). You are free to choose any of these options, but in any case you must document your approach and explain all of your steps.