The exam will cover Chapters 12 and 13 of the text. There will be some straight forward computational problems like in the Webwork or text exercises. There will also be a few true/false questions. Other problems will involve more thought, but use the concepts we've discussed. Here is a sample of such problems that have been given in the past.

1. Let $\mathbf{a} = (x+y)\mathbf{i} + 2\mathbf{j} + y\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + (4x+y+1)\mathbf{j} + 4\mathbf{k}$. Find the relationship between x and y such that $\mathbf{a}$ and $\mathbf{b}$ are orthogonal.

2. Find a vector function that represents the curve of intersection of the paraboloid $z = 3x^2 + y^2$ and the parabolic cylinder $y = x^2$.

3. Show that the line given by $x = t$, $y = 3t - 2$, $z = -t$ intersects the plane $x + y + z = 1$ and find the point of intersection.

4. Sketch the portion of the plane $x + y + z = 1$ that lies in the first octant.

5. Suppose that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are three distinct unit vectors in $\mathbb{R}^3$ which satify the following two conditions: $\mathbf{b} \times \mathbf{c} \neq \mathbf{0}$ and $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{0}$. Show that $\mathbf{a}$ is perpendicular to both $\mathbf{b}$ and $\mathbf{c}$. (Hint: use the formula for $\|\mathbf{u} \times \mathbf{v}\|$.)

6. Find the angle θ between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$.

7. Find the symmetric equation for the line of intersection between the two planes $x + y + z = 1$ and $x - 2y + 3z = 1$.

8. Let $\mathbf{r}(t) = \sin(2t)\mathbf{i} + 3t\mathbf{j} + \cos(2t)\mathbf{k}$ where $-\pi \leq t \leq \pi$. Find $\mathbf{r}'(t)$, $\mathbf{r}''(t)$, $\mathbf{T}(0)$ and $\mathbf{r}'(t) \times \mathbf{r}''(t)$.

9. Let $y = x^2$ be a parabola in the xy-plane parametrized by $\mathbf{r}(t) =< t, t^2, 0 >$. Find the unit tangent vector at the origin.

10. Determine whether the following curves are smooth.
    (a) $\mathbf{r}(t) =< t^3, t^4, t^5 >$
    (b) $\mathbf{r}(t) =< t^3 + t, t^4, t^5 >$

11. Evaluate the integrals.
    (a) $\displaystyle\int_0^1 \left(\frac{4}{1+t^2}\mathbf{j} + \frac{2t}{1+t^2}\mathbf{k} \right) dt$
    (b) $\displaystyle\int ((\cos \pi t)\mathbf{i} + (\sin \pi t)\mathbf{j} + t\mathbf{k})\, dt$
    (c) $\displaystyle\int_1^4 \left(\sqrt{t}\mathbf{i} + te^{-t}\mathbf{j} + \frac{1}{t^2}\mathbf{k} \right) dt$

12. A particle moves along a curve defined by
$$\mathbf{r}(t) =< 2\cos(2t), 2\sin(2t), 3t > .$$
    (a) Reparametrize the curve in terms of arc length s.
    (b) Find a unit tangent vector to the curve in terms of s.
    (c) Show that the curve has constant curvature.

13. For the curve given by $\mathbf{r}(t) = \left\langle \frac{1}{3}t^3,\ \frac{1}{2}t^2,\ t \right\rangle$, find $\mathbf{T}(t)$.

14. For the curve given by $\mathbf{r}(t) = \langle \cos t^2,\ 7, \sin t^2 \rangle$ with $t \geq 0$, find

    (a) $\mathbf{T}(t)$, (b) $\mathbf{N}(t)$, (c) $\mathbf{B}(t)$

15. Let $\mathbf{r}(t) = \sin(2t)\mathbf{i} + 3t\mathbf{j} + \cos(2t)\mathbf{k}$ where $-\pi \leq t \leq \pi$ be the position vector of a particle at time t.

    (a) Show that the velocity and acceleration vectors are always perpendicular.

    (b) Is there a time t when the position and velocity vectors are perpendicular? If so, find all such time(s) t and the corresponding position(s) of the particle.

16. Let $y = x^2$ be a parabola in the xy-plane parametrized by $\mathbf{r}(t) =< t, t^2, 0 >$. Find the unit tangent vector, the unit normal vector and the binormal vector at the origin.

17. Let C be the circle of radius 3 centered at the origin in the xy-plane.

    (a) Find a parametrization for C in the form $x = f(t)$, $y = g(t)$ for t in some interval and use it to obtain the position vector $\mathbf{r}(t)$ for C.

    (b) Give the arc length parametrization $\mathbf{r}(s)$ of this curve, starting at the point $(3, 0)$.

    (c) Find the curvature κ of C.

18. A moving particle starts at an initial position $\mathbf{r}(0) = (4, 2, 0)$ with initial velocity $\mathbf{v}(0) = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$. Its acceleration is $\mathbf{a}(t) = e^{2t}\mathbf{j} - 20\mathbf{k}$.

    (a) Find the velocity vector $\mathbf{v}(t)$ for the particle.

    (b) Find the position vector $\mathbf{r}(t)$ for the particle.