THERE MAY BE TYPOS in these solutions. Please let me know if you find any.

1. Let $\mathbf{a} = (x+y)\mathbf{i} + 2\mathbf{j} + y\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + (4x+y+1)\mathbf{j} + 4\mathbf{k}$. Find the relationship between x and y such that $\mathbf{a}$ and $\mathbf{b}$ are orthogonal.

$$\mathbf{a} \text{ and } \mathbf{b} \text{ are orthogonal } \iff \mathbf{a}\cdot\mathbf{b} = 0$$

$$\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= 3(x+y) + 2(4x+y+1) + 4y = 0\\ &\implies 3x + 3y + 8x + 2y + 2 + 4y = 0\\ &\implies 11x + 9y = -2\end{aligned}$$

$$\boxed{11x + 9y = -2 \text{ (or some equivalent equation for this line)}}$$

2. Find a vector function that represents the curve of intersection of the paraboloid $z = 3x^2 + y^2$ and the parabolic cylinder $y = x^2$.

$$C_1 : z = 3x^2 + y^2, \qquad C_2 : y = x^2$$

By plugging in the equation for C_2 into C_1, we get

$$z = 3y + y^2$$

So if we let $x = t$ for $t \in \mathbb{R}$, then $y = t^2$ and $z = 3t^2 + t^4$ to get

$$\boxed{\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + (3t^2 + t^4)\mathbf{k} = \langle t,\ t^2,\ 3t^2 + t^4\rangle}$$

3. Show that the line given by $x = t$, $y = 3t - 2$, $z = -t$ intersects the plane $x + y + z = 1$ and find the point of intersection.

Plug in the parametric equations for the line into the equation of the plane to get:

$$x + y + z = t + 3t - 2 - t = 1 \implies 3t - 2 = 1 \implies t = 1$$

So the line intersects the plane when $t = 1$, or more specifically, at the point $x = 1$, $y = 1$ and $z = -1$

$$\boxed{\text{Point of intersection: } (1, 1, -1)}$$

4. Sketch the portion of the plane $x + y + z = 1$ that lies in the first octant.

To sketch, try and find intercepts.

$$\begin{aligned}&x\text{-int: when } y = z = 0 \implies x = 1 \implies (1,0,0)\\ &y\text{-int: when } x = z = 0 \implies y = 1 \implies (0,1,0)\\ &z\text{-int: when } x = y = 0 \implies z = 1 \implies (0,0,1)\end{aligned}$$

Since 3 points determine a plane, can use these points to draw the sketch (sketch not shown)

5. TYPO IN HINT FIXED. Suppose that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are three distinct unit vectors in $\mathbb{R}^3$ which satify the following two conditions: $\mathbf{b}\times\mathbf{c} \neq \mathbf{0}$ and $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = \mathbf{0}$. Show that $\mathbf{a}$ is perpendicular to both $\mathbf{b}$ and $\mathbf{c}$. (Hint: use the formula for $\|\mathbf{u}\times\mathbf{v}\|$.)

$$\mathbf{b}\times\mathbf{c} \neq \mathbf{0} \implies \|\mathbf{b}\times\mathbf{c}\| = \|\mathbf{b}\|\ \|\mathbf{c}\| \sin\theta_1 \neq 0$$

Since $\mathbf{b}$ and $\mathbf{c}$ are unit vectors, $\|\mathbf{b}\| = \|\mathbf{c}\| = 1$ and thus $\|\mathbf{b}\times\mathbf{c}\| = \sin\theta_1$. Thus $\|\mathbf{b}\times\mathbf{c}\| = \sin\theta_1 \neq 0$.

We are given that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{0}$. But $\|\mathbf{a} \times (\mathbf{b} \times \mathbf{c})\| = \|\mathbf{a}\| \, \|\mathbf{b} \times \mathbf{c}\| \sin\theta_2$ and since $\|\mathbf{a}\| = 1$, we get

$$0 = \|\mathbf{a} \times (\mathbf{b} \times \mathbf{c})\| = \sin\theta_1 \sin\theta_2$$

The only way for this to happen is for $\sin\theta_2 = 0$, in other words, θ_2, the angle between $\mathbf{a}$ and $\mathbf{b} \times \mathbf{c}$ is 0 or π. So $\mathbf{a}$ is parallel to $\mathbf{b} \times \mathbf{c}$. Thus $\mathbf{a}$ is orthogonal to $\mathbf{b}$ and $\mathbf{c}$ since $\mathbf{b} \times \mathbf{c}$ is orthogonal to both $\mathbf{b}$ and $\mathbf{c}$.

6. Find the angle θ between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$.

   Look at the normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$.

$$\mathbf{n}_1 =< 1, 1, 1 > \qquad \mathbf{n}_2 =< 1, -2, 3 > \qquad \mathbf{n}_1 \cdot \mathbf{n}_2 = \|\mathbf{n}_1\| \, \|\mathbf{n}_2\| \cos\theta$$

$$\begin{aligned} \Longrightarrow \cos\theta &= \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{\|\mathbf{n}_1\| \, \|\mathbf{n}_2\|} = \frac{1 - 2 + 3}{\sqrt{3} \cdot \sqrt{14}} = \frac{2}{\sqrt{42}} \\ \Longrightarrow \theta &= \cos^{-1}\left(\frac{2}{\sqrt{42}}\right) \text{ (this is the exact value)} \\ &\approx \boxed{1.257 \text{ radians or } 72^\circ} \end{aligned}$$

7. Find the symmetric equation for the line of intersection between the two planes $x + y + z = 1$ and $x - 2y + 3z = 1$.

   Try setting $z = 0$ into both plane equations: $\begin{matrix} P_1 : x + y = 1 \\ P_2 : x - 2y = 1 \end{matrix} \Longrightarrow 3y = 0 \Longrightarrow y = 0 \Longrightarrow x = 1$

   So a point on the line is $(1, 0, 0)$ Now we need the direction vector for the line. The normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are perpendicular to vectors and lines in planes P_1 and P_2, respectively. Thus the normal vectors for both planes are perpendicular to the line of intersection; i.e., the line is orthogonal to both normal vectors. Thus, the we can use $\mathbf{n}_1 \times \mathbf{n}_2$ as the direction vector for the line.

$$\begin{aligned} \mathbf{n}_1 \times \mathbf{n}_2 &=< 1, 1, 1 > \times < 1, -2, 3 > \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} \\ &= \mathbf{i}(3 - (-2)) - \mathbf{j}(3 - 1) + \mathbf{k}(-2 - 1) = 5\mathbf{i} - 2\mathbf{j} - 3\mathbf{k} \end{aligned}$$

$$\boxed{\frac{x - 1}{5} = \frac{y}{-2} = \frac{z}{-3}}$$

8. Let $\mathbf{r}(t) = \sin(2t)\mathbf{i} + 3t\mathbf{j} + \cos(2t)\mathbf{k}$ where $-\pi \le t \le \pi$. Find $\mathbf{r}'(t)$, $\mathbf{r}''(t)$, $\mathbf{T}(0)$ and $\mathbf{r}'(t) \times \mathbf{r}''(t)$.

$$\begin{aligned} \mathbf{r}'(t) &= 2\cos(2t)\mathbf{i} + 3\mathbf{j} - 2\sin(2t)\mathbf{k} \\ \mathbf{r}''(t) &= -4\sin(2t)\mathbf{i} + 0\mathbf{j} - 4\cos(2t)\mathbf{k} \\ \mathbf{T}(0) &= \frac{\mathbf{r}'(0)}{\|\mathbf{r}'(0)\|}, \qquad \mathbf{r}'(0) =< 2, 3, 0 >, \qquad \|\mathbf{r}'(0)\| = \sqrt{13} \\ \mathbf{T}(0) &= \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}, 0 \right\rangle \end{aligned}$$

$$\begin{aligned}
\mathbf{r}'(t)\times\mathbf{r}''(t) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2\cos(2t) & 3 & -2\sin(2t) \\ -4\sin(2t) & 0 & -4\cos(2t) \end{vmatrix} \\
&= \mathbf{i}(-12\cos(2t)) - \mathbf{j}(-8\cos^2(2t) - 8\sin^2(2t)) + \mathbf{k}(12\sin(2t)) \\
&= -12\cos(2t)\mathbf{i} + 8\mathbf{j} + 12\sin(2t)\mathbf{k}
\end{aligned}$$

$$\boxed{\begin{array}{ll} \mathbf{r}'(t) = 2\cos(2t)\mathbf{i} + 3\mathbf{j} - 2\sin(2t)\mathbf{k} & \mathbf{r}''(t) = -4\sin(2t)\mathbf{i} + 0\mathbf{j} - 4\cos(2t)\mathbf{k} \\ \mathbf{T}(0) = \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}, 0 \right\rangle & \mathbf{r}'(t)\times\mathbf{r}''(t) = -12\cos(2t)\mathbf{i} + 8\mathbf{j} + 12\sin(2t)\mathbf{k} \end{array}}$$

9. Let $y = x^2$ be a parabola in the xy-plane parametrized by $\mathbf{r}(t) =< t, t^2, 0 >$. Find the unit tangent vector at the origin.

$$\begin{aligned}
\mathbf{r}'(t) &=< 1, 2t, 0 > \\
\mathbf{r}'(0) &=< 1, 0, 0 > \\
\|\mathbf{r}'(0)\| &= 1
\end{aligned}$$

$$\boxed{\mathbf{T}(0) =< 1, 0, 0 >}$$

10. Determine whether the following curves are smooth.

(a) $\mathbf{r}(t) =< t^3, t^4, t^5 >$

$$\begin{aligned}
\mathbf{r}'(t) &=< 3t^2, 4t^3, 5t^4 > \text{ is continuous for any } t \\
&= \mathbf{0} \text{ when } t = 0
\end{aligned}$$

$$\boxed{\mathbf{r}(t) \text{ is not smooth}}$$

(b) $\mathbf{r}(t) =< t^3 + t, t^4, t^5 >$

$$\begin{aligned}
\mathbf{r}'(t) &=< 3t^2 + 1, 4t^3, 5t^4 > \text{ is continuous for any } t \\
&\neq \mathbf{0} \text{ for any } t
\end{aligned}$$

$$\boxed{\mathbf{r}(t) \text{ is smooth}}$$

11. Evaluate the integrals.

(a) $\displaystyle\int_0^1 \left(\frac{4}{1+t^2}\mathbf{j} + \frac{2t}{1+t^2}\mathbf{k} \right) dt$

$$\begin{aligned}
\int_0^1 \left(\frac{4}{1+t^2}\mathbf{j} + \frac{2t}{1+t^2}\mathbf{k} \right) dt &= \left(\int_0^1 \frac{4}{1+t^2}\, dt \right)\mathbf{j} + \left(\int_0^1 \frac{2t}{1+t^2}\, dt \right)\mathbf{k} \\
&= \left(4\tan^{-1} t\Big|_0^1 \right)\mathbf{j} + \left(\ln(1+t^2)\Big|_0^1 \right)\mathbf{k} \\
&= \boxed{\pi\mathbf{j} + (\ln 2)\mathbf{k}}
\end{aligned}$$

(b) $\int ((\cos \pi t)\mathbf{i} + (\sin \pi t)\mathbf{j} + t\mathbf{k})\ dt$

$$\begin{aligned}\int ((\cos \pi t)\mathbf{i} + (\sin \pi t)\mathbf{j} + t\mathbf{k})\ dt &= \left(\int \cos \pi t\, dt\right)\mathbf{i} + \left(\int \sin \pi t\, dt\right)\mathbf{j} + \left(\int t\, dt\right)\mathbf{k} \\ &= \boxed{\left\langle \frac{1}{\pi}\sin \pi t, -\frac{1}{\pi}\cos \pi t, \frac{1}{2}t^2 \right\rangle}\end{aligned}$$

(c) $\int_1^4 \left(\sqrt{t}\mathbf{i} + te^{-t}\mathbf{j} + \frac{1}{t^2}\mathbf{k}\right)\ dt$

$$\begin{aligned}\int_1^4 \left(\sqrt{t}\mathbf{i} + te^{-t}\mathbf{j} + \frac{1}{t^2}\mathbf{k}\right)\ dt &= \left(\int_1^4 t^{1/2}\ dt\right)\mathbf{i} + \left(\int_1^4 te^{-t}\ dt\right)\mathbf{j} + \left(\int_1^4 t^{-2}\ dt\right)\mathbf{k} \\ &= \left(\frac{2}{3}t^{3/2}\Big|_1^4\right)\mathbf{i} + \left(-te^{-t}\Big|_1^4 + \int_1^4 e^{-t}\ dt\right)\mathbf{j} + \left(-t^{-1}\Big|_1^4\right)\mathbf{k} \\ &= \left(\frac{16}{3} - \frac{2}{3}\right)\mathbf{i} + \left(-4e^{-4} + e^{-1} - (e^{-t}\Big|_1^4)\right)\mathbf{j} + \left(-\frac{1}{4} + 1\right)\mathbf{k} \\ &= \boxed{\frac{14}{3}\mathbf{i} + (-5e^{-4} + 2e^{-1})\mathbf{j} + \frac{3}{4}\mathbf{k}}\end{aligned}$$

12. A particle moves along a curve defined by

$$\mathbf{r}(t) = < 2\cos(2t), 2\sin(2t), 3t > .$$

(a) Reparametrize the curve in terms of arc length s.

$$\begin{aligned}\mathbf{r}'(t) &= < -4\sin(2t), 4\cos(2t), 3 > \\ \|\mathbf{r}'(t)\| &= \sqrt{16\sin^2(2t) + 16\cos^2(2t) + 9} \\ &= \sqrt{16+9} = 5 \\ s &= \int_0^t \|\mathbf{r}'(u)\|\, du = \int_0^t 5\, du = 5t \\ s &= 5t \Longrightarrow t = \frac{s}{5} \\ \therefore \mathbf{r}(t(s)) = \mathbf{r}(s) &= \boxed{\left\langle 2\cos\left(\frac{2s}{5}\right), 2\sin\left(\frac{2s}{5}\right), \frac{3s}{5} \right\rangle}\end{aligned}$$

(b) Find a unit tangent vector to the curve in terms of s.

$$\begin{aligned}\mathbf{r}'(s) &= \left\langle -\frac{4}{5}\sin\left(\frac{2s}{5}\right), \frac{4}{5}\cos\left(\frac{2s}{5}\right), \frac{3}{5} \right\rangle \\ \|\mathbf{r}'(s)\| &= \sqrt{\frac{16}{25}\sin^2\left(\frac{2s}{5}\right) + \frac{16}{25}\cos^2\left(\frac{2s}{5}\right) + \frac{9}{25}} \\ &= \sqrt{\frac{16}{25} + \frac{9}{25}} = 1 \\ \mathbf{T}(s) &= \frac{\mathbf{r}'(s)}{\|\mathbf{r}'(s)\|} = \boxed{\left\langle -\frac{4}{5}\sin\left(\frac{2s}{5}\right), \frac{4}{5}\cos\left(\frac{2s}{5}\right), \frac{3}{5} \right\rangle}\end{aligned}$$

(c) Show that the curve has constant curvature.

$$\kappa = \|\mathbf{T}'(s)\| \text{ (using the defn, not our usual formula)}$$

$$\mathbf{T}'(s) = \left\langle -\frac{8}{25}\cos(2s/5),\ -\frac{8}{25}\sin(2s/5),\ 0\right\rangle$$

$$\kappa = \|\mathbf{T}'(s)\| = \sqrt{\frac{64}{625}\cos^2(2s/5) + \frac{64}{625}\sin^2(2s/5)}$$

$$= \sqrt{\frac{64}{625}} = \frac{8}{25} \text{ no matter what } s \text{ is so curvature is constant}$$

13. For the curve given by $\mathbf{r}(t) = \left\langle \frac{1}{3}t^3,\ \frac{1}{2}t^2,\ t\right\rangle$, find $\mathbf{T}(t)$.

$$\mathbf{r}'(t) = \langle t^2,\ t,\ 1\rangle, \qquad \|\mathbf{r}'(t)\| = \sqrt{t^4 + t^2 + 1}$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} = \boxed{\frac{1}{\sqrt{t^4+t^2+1}}\langle t^2,\ t,\ 1\rangle}$$

14. For the curve given by $\mathbf{r}(t) = \langle \cos t^2,\ 7, \sin t^2\rangle$ with $t \geq 0$, find

(a) $\mathbf{T}(t)$,

$$\mathbf{r}'(t) =< -2t\sin t^2,\ 0, 2t\cos t^2 >$$

$$\|\mathbf{r}'(t)\| = \sqrt{4t^2\sin^2 t^2 + 4t^2\cos^2 t^2} = 2t \text{ (since } t \geq 0)$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} = \boxed{< -\sin t^2, 0, \cos t^2 >}$$

(b) $\mathbf{N}(t)$,

$$\mathbf{T}'(t) =< -2t\cos t^2, 0, -2t\sin t^2 >$$

$$\|\mathbf{T}'(t)\| = \sqrt{4t^2\cos^2 t^2 + 4t^2\sin^2 t^2} = 2t$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|} = \boxed{< -\cos t^2, 0, -\sin t^2 >}$$

(c) $\mathbf{B}(t)$

$$\mathbf{B}(t) = \mathbf{T} \times \mathbf{N}$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\sin t^2 & 0 & \cos t^2 \\ -\cos t^2 & 0 & -\sin t^2 \end{bmatrix}$$

$$= \mathbf{i}(0) - \mathbf{j}(\sin^2 t^2 + \cos^2 t^2) + \mathbf{k}(0) = \boxed{< 0, -1, 0 >}$$

15. Let $\mathbf{r}(t) = \sin(2t)\mathbf{i} + 3t\mathbf{j} + \cos(2t)\mathbf{k}$ where $-\pi \leq t \leq \pi$ be the position vector of a particle at time t.

(a) Show that the velocity and acceleration vectors are always perpendicular.

$$\mathbf{v}(t) = \mathbf{r}'(t) =< 2\cos(2t),\ 3,\ -2\sin(2t) >$$

$$\mathbf{a}(t) = \mathbf{v}'(t) =< -4\sin(2t),\ 0,\ -4\cos(2t) >$$

$$\mathbf{a}(t)\cdot\mathbf{v}(t) = -8\sin(2t)\cos(2t) + 8\cos(2t)\sin(2t) = 0$$

Since the dot product of the velocity and acceleration vectors is always 0, they are always perpendicular.

(b) Is there a time t when the position and velocity vectors are perpendicular? If so, find all such time(s) t and the corresponding position(s) of the particle.

$$\begin{aligned}\mathbf{r}(t)\cdot\mathbf{v}(t) &= 2\sin(2t)\cos(2t)+9t-2\cos(2t)\sin(2t)\\ &= 9t = 0 \iff t=0\\ \mathbf{r}(0) &= <\sin(0),\,0,\cos(0)> = <0,0,1>\end{aligned}$$

They are perpendicular only when $t=0$. The position of the particle at this time is $<0,0,1>$.

16. Let $y=x^2$ be a parabola in the xy-plane parametrized by $\mathbf{r}(t) =< t,t^2,0 >$. Find the unit tangent vector, the unit normal vector and the binormal vector at the origin.

$$\mathbf{r}'(t) =< 1,2t,0 > \qquad \|\mathbf{r}'(t)\| = \sqrt{1+4t^2}, \qquad \mathbf{T}(t) = \left\langle \frac{1}{\sqrt{1+4t^2}}, \frac{2t}{\sqrt{1+4t^2}}, 0\right\rangle$$

$$< 0,0,0 > \iff t=0, \text{ so } \boxed{\mathbf{T}(0) =< 1,0,0 >}$$

$$\begin{aligned}\mathbf{T}'(t) &= \left\langle -\frac{1}{2}(1+4t^2)^{-3/2}(8t),\ \frac{2(1+4t^2)^{1/2}-2t\frac{1}{2}(1+4t^2)^{-1/2}(8t)}{1+4t^2},\ 0\right\rangle\\ &= \left\langle \frac{-4t}{(1+4t^2)^{3/2}}, \frac{2(1+4t^2)^{1/2}-\frac{8t^2}{(1+4t^2)^{1/2}}}{1+4t^2}, 0\right\rangle = \left\langle \frac{-4t}{(1+4t^2)^{3/2}}, \frac{2(1+4t^2)-8t^2}{(1+4t^2)^{3/2}}, 0\right\rangle\\ &= \left\langle \frac{-4t}{(1+4t^2)^{3/2}}, \frac{2}{(1+4t^2)^{3/2}}, 0\right\rangle \implies \mathbf{T}'(0) =< 0,2,0 >, \quad \|\mathbf{T}'(0)\| = \sqrt{0+4+0} = 2\\ \mathbf{N}(0) &= \frac{\mathbf{T}'(0)}{\|\mathbf{T}'(0)\|} \implies \boxed{\mathbf{N}(0) =< 0,1,0 >}\\ \mathbf{B}(0) &= \mathbf{T}(0)\times\mathbf{N}(0) = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 0 & 0\\ 0 & 1 & 0\end{vmatrix} = 0\mathbf{i}-0\mathbf{j}+1\mathbf{k} = \mathbf{k} \implies \boxed{\mathbf{B}(0) =< 0,0,1 >}\end{aligned}$$

17. Let C be the circle of radius 3 centered at the origin in the xy-plane.

(a) Find a parametrization for C in the form $x=f(t)$, $y=g(t)$ for t in some interval and use it to obtain the position vector $\mathbf{r}(t)$ for C.

$$x^2+y^2=3^2 \qquad \implies x = 3\cos t,\ y = 3\sin t,\ t\in[0,2\pi]$$

$$\boxed{\mathbf{r}(t) =< 3\cos t, 3\sin t >}$$

(b) Give the arc length parametrization $\mathbf{r}(s)$ of this curve, starting at the point $(3,0)$.

$$\begin{aligned}(3,0) &\implies t=0\\ \mathbf{r}'(t) &=< -3\sin t, 3\cos t >\\ \|\mathbf{r}'(t)\| &= \sqrt{9\sin^2 t+9\cos^2 t} = 3\\ s &= \int_0^t \|\mathbf{r}'(u)\|\,du = \int_0^t 3\,du = 3t\\ t &= s/3\end{aligned}$$

$$\boxed{\mathbf{r}(s) =< 3\cos(s/3),\ 3\sin(s/3) >}$$

(c) Find the curvature κ of C.

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$
$$\mathbf{T}(t) = \frac{1}{3} < -3\sin t,\ 3\cos t >$$
$$=< -\sin t,\ \cos t >$$
$$\mathbf{T}'(t) =< -\cos t,\ -\sin t >$$
$$\|\mathbf{T}'(t)\| = \sqrt{\cos^2 t + \sin^2 t} = 1,$$
$$\|\mathbf{r}'(t)\| = 3$$
$$\Longrightarrow \kappa = \boxed{\frac{1}{3}}$$

OR $\kappa = \|\mathbf{T}'(s)\|$

$$\mathbf{r}'(s) =< -\sin(s/3),\ \cos(s/3) >$$
$$\|\mathbf{r}'(s)\| = \sqrt{\sin^2(s/3) + \cos^2(s/3)} = 1$$
$$\mathbf{T}(s) =< -\sin(s/3),\ \cos(s/3) >$$
$$\mathbf{T}'(s) = \left\langle -\frac{1}{3}\cos(s/3),\ -\frac{1}{3}\sin(s/3) \right\rangle$$
$$\Longrightarrow \kappa = \sqrt{\frac{1}{9}\cos^2(s/3) + \frac{1}{9}\sin^2(s/3)}$$
$$= \sqrt{\frac{1}{9}} = \boxed{\frac{1}{3}}$$

18. A moving particle starts at an initial position $\mathbf{r}(0) = (4, 2, 0)$ with initial velocity $\mathbf{v}(0) = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$. Its acceleration is $\mathbf{a}(t) = e^{2t}\mathbf{j} - 20\mathbf{k}$.

(a) Find the velocity vector $\mathbf{v}(t)$ for the particle.

$$\mathbf{v}(t) = \int \mathbf{a}(t)\, dt + \mathbf{C}, \quad \mathbf{C} =< c_1, c_2, c_3 >$$
$$= \left\langle 0,\ \frac{1}{2}e^{2t},\ -20t \right\rangle + < c_1, c_2, c_3 >$$
$$\mathbf{v}(0) =< 2, 3, -1 >= \left\langle c_1, \frac{1}{2} + c_2, c_3 \right\rangle \Longrightarrow c_1 = 2 \quad c_2 = \frac{5}{2}, \quad c_3 = -1$$
$$\mathbf{v}(t) = \boxed{\left\langle 2,\ \frac{e^{2t}}{2} + \frac{5}{2},\ -20t - 1 \right\rangle}$$

(b) Find the position vector $\mathbf{r}(t)$ for the particle.

$$\mathbf{r}(t) = \int \mathbf{v}(t)\ dt + \mathbf{D}, \quad \mathbf{D} =< d_1, d_2, d_3 >$$
$$\mathbf{r}(t) = \left\langle 2t + d_1,\ \frac{e^{2t}}{4} + \frac{5}{2}t + d_2,\ -10t^2 - t + d_3 \right\rangle$$
$$\mathbf{r}(0) =< 4, 2, 0 >= \left\langle d_1,\ \frac{1}{4} + d_2,\ d_3 \right\rangle \Longrightarrow d_1 = 4 \qquad d_2 + \frac{7}{4} \qquad d_3 = 0$$
$$\mathbf{r}(t) = \boxed{\left\langle 2t + 4,\ \frac{e^{2t}}{4} + \frac{5}{2}t + \frac{7}{4},\ -10t^2 - t \right\rangle}$$