The exam is on Thursday, October 30 and will cover Chapter 14. Here is a sample of problems that have been given in the past. All problems will be computational in nature like your webwork, homework exercises or the problems below; there will be no true/false questions on this exam.

1. Consider the function
$$f(x,y)=\sqrt{y-x}.$$
   (a) Sketch the domain of f.
   (b) Sketch the level curves $f(x,y)=k$ for $k=0,1,2,3$.
   (c) Find f_x.
   (d) Find f_y.
   (e) Find the equation of the tangent plane to f at the point $(2,6,2)$.

2. Show that $\lim_{(x,y)\to(0,0)} \dfrac{3xy}{x^2+y^2}$ doesn't exist.

3. Given $u(t,x)=e^{-\alpha^2k^2t}\sin(kx)$, evaluate $\dfrac{\partial u}{\partial t}-\alpha^2\dfrac{\partial^2 u}{\partial x^2}$, simplifying as much as possible.

4. Given $x-z=\arctan(yz)$, find
   (a) $\dfrac{\partial z}{\partial x}$ (b) $\dfrac{\partial z}{\partial y}$

5. Consider the surface given implicitly by $xy+yz+xz=7$. Find
   (a) $\dfrac{\partial z}{\partial x}$ (b) $\dfrac{\partial z}{\partial y}$

6. Suppose $u=xy+yz+xz$, $x=st$, $y=e^{st}$ and $z=t^2$.
   (a) Find $\dfrac{\partial u}{\partial s}$ at the point $(s,t)=(0,1)$.
   (b) Find $\dfrac{\partial u}{\partial t}$ at the point $(s,t)=(0,1)$.

7. Suppose $z=2xy+3y^2+ye^x$, $x=r\cos\theta$ and $y=r\sin\theta$.
   (a) Find $\dfrac{\partial z}{\partial r}$ at the point $(r,\theta)=(2,\pi)$.
   (b) Find $\dfrac{\partial z}{\partial \theta}$ at the point $(r,\theta)=(2,\pi)$.

8. Verify that the function $f(x,y)=\ln\sqrt{x^2+y^2}$ is a solution to Laplace's equation
$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0.$$

9. Calculate the limits, or show that they don't exist.
   (a) $\lim_{(x,y,z)\to(4,1,-2)} e^{x^2z}\cos(2y+z)$
   (b) $\lim_{(x,y)\to(0,0)} \dfrac{5x^4y^2}{x^8+y^8}$

10. Find all second partial derivatives of $f(x,y)=\ln(3x+5y)$.

11. Find the linear approximation of $f(x, y) = \ln(x - 3y)$ at $(7, 2)$ and use it to approximate $f(6.9, 2.06)$.

12. The pressure, volume and temperature of a mole of an ideal gas are related by the equation $PV = 8.31T$, where P is measured in kilopascals, V in liters, and T in kelvins. Use differentials to find the approximate change in the pressure if the volume increases from 12 L to 12.3 L and the temperature decreases from 310 K to 305 K.

13. Consider the function $f(x, y) = 3x^2 - xy + y^3$.
    (a) Find the rate of change of f at $(1, 2)$ in the direction of $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$.
    (b) From the point $(1, 2)$, in what direction does f decrease the most? Give your answer as a unit vector. What is this maximum rate of decrease?
    (c) From the point $(1, 2)$, in what direction does f increase the most? Give your answer as a unit vector. What is this maximum rate of increase?
    (d) From the point $(1, 2)$, in what direction(s) is the rate of change of f equal to zero? Give your answer(s) as unit vector(s).

14. Suppose $f(x, y)$ is a function such that $\nabla f(2, 4)$ has norm of 5. Is there a direction $\mathbf{u}$ such that the directional derivative $D_{\mathbf{u}} f(2, 4) = 7$? Explain your answer.

15. Consider the ellipsoid $x^2 + 4y^2 = 169 - 9z^2$ and the point $P(3, 2, 4)$ on the ellipsoid.
    (a) Find the equation of the tangent plane to the ellipsoid at the point P.
    (b) Find the parametric equations for the normal line to the ellipsoid at the point P.

16. Let $f(x, y) = x^2 + \dfrac{y^2}{2} + x^2 y$.
    (a) Find all critical points of f.
    (b) Apply the second derivative test to each of them, and write down the result of the test.

17. Consider the function $f(x, y) = y^2 - x^2 + x^4$. Find and classify (as maxima, minima or saddles) the critical points of f, showing all work.

18. Find the maximum of $f(x, y) = xy$ restricted to the curve $(x+1)^2 + y^2 = 1$. Give both the coordinates of the point and the value of f.

19. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is a constant, C.

20. Find the absolute maximum and minimum of $f(x, y) = x^2 + xy + y^2$ over the disk $\{(x, y) \mid x^2 + y^2 \le 9\}$.