The Many Lessons in Fractals

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MAA Mathfest
Washington, D.C.

Course

Programming in Mathematics (MA302): 3 credit course

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- Required of all mathematics majors; elective for statistics majors

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- Assignment and project based

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- Co-requisite: intro. to linear algebra
- Assignment and project based
- Currently use MATLAB

Linear Transformations

Given vertices of a polygon, draw original and transformed shapes

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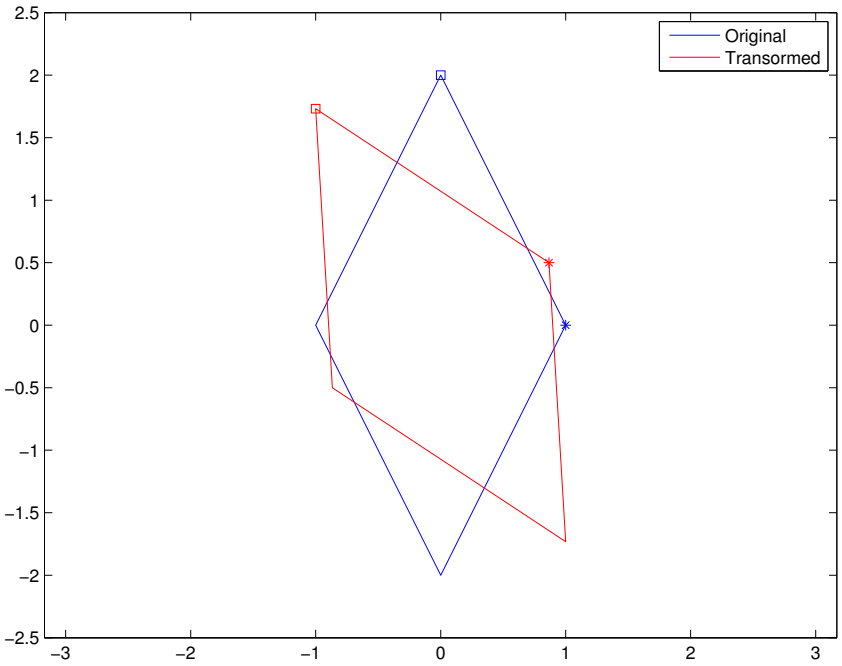
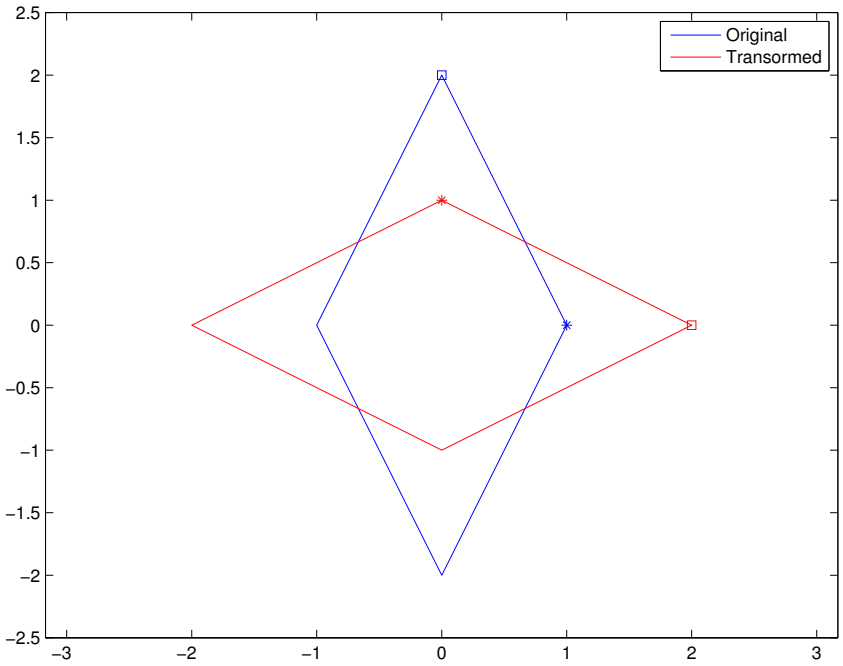
Given vertices of a polygon, draw original and transformed shapes

- Rotation
- Reflection
- Scaling (contraction and dilation)
- Composition
- A given 2×2 matrix

Linear Transformations



(a) Rotation



(b) Reflection

Affine Transoformations and Fern Fractals

Upload matrices for affine transformations, then apply to polygon.

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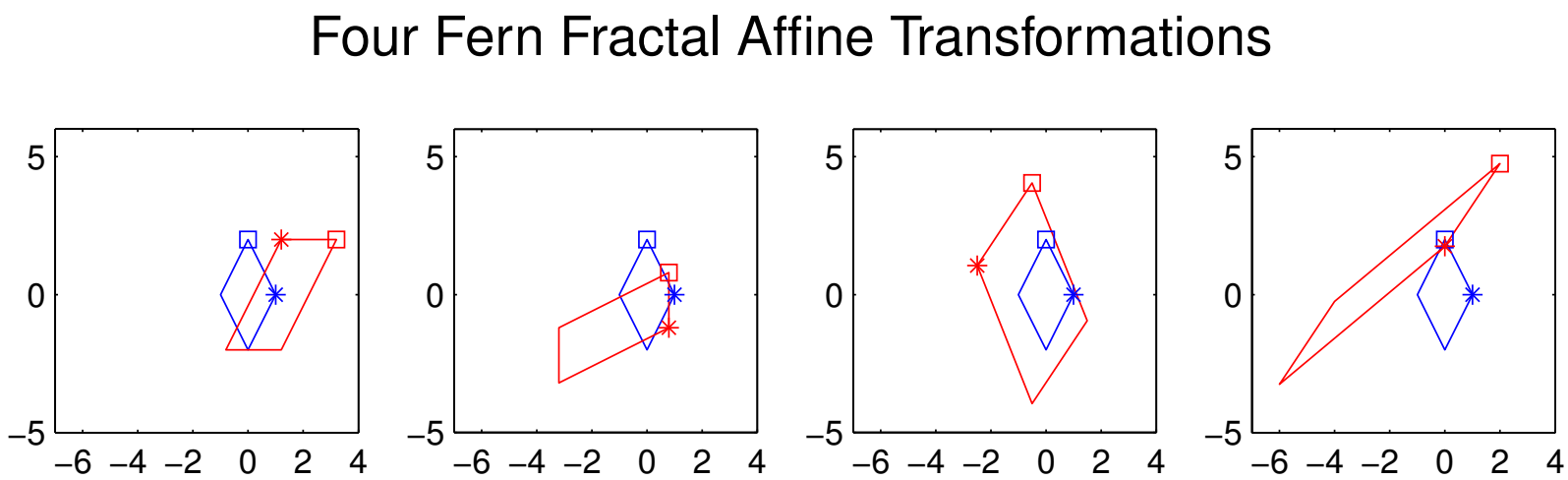

Fern Fractals and Code Improvement

Create three fern fractal functions

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(Good for control statements, loops, random numbers)

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1. Traditional way: plot each point as it is calculated

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2. Store each point in matrix, then plot the matrix

Fern Fractals and Code Improvement

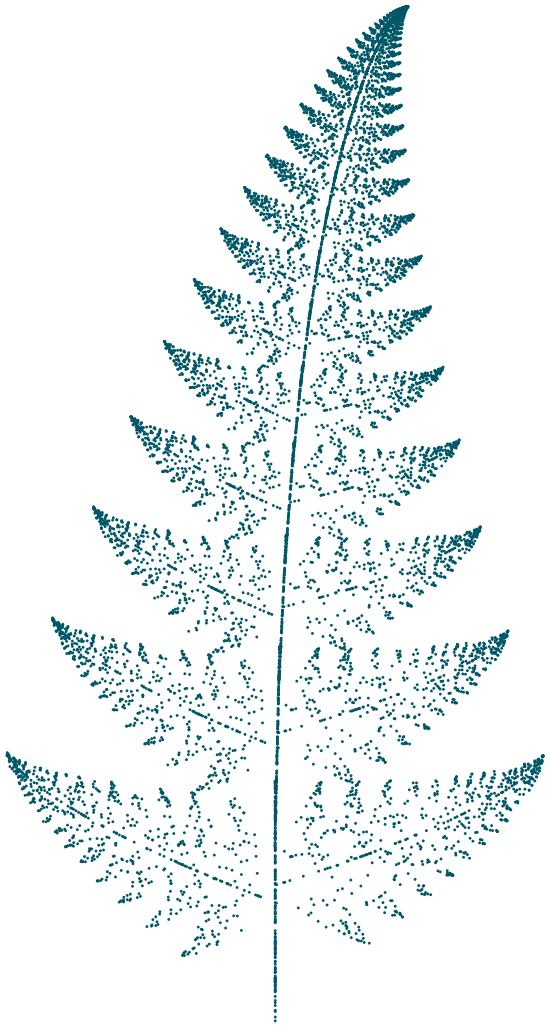
Create three fern fractal functions

(Good for control statements, loops, random numbers)

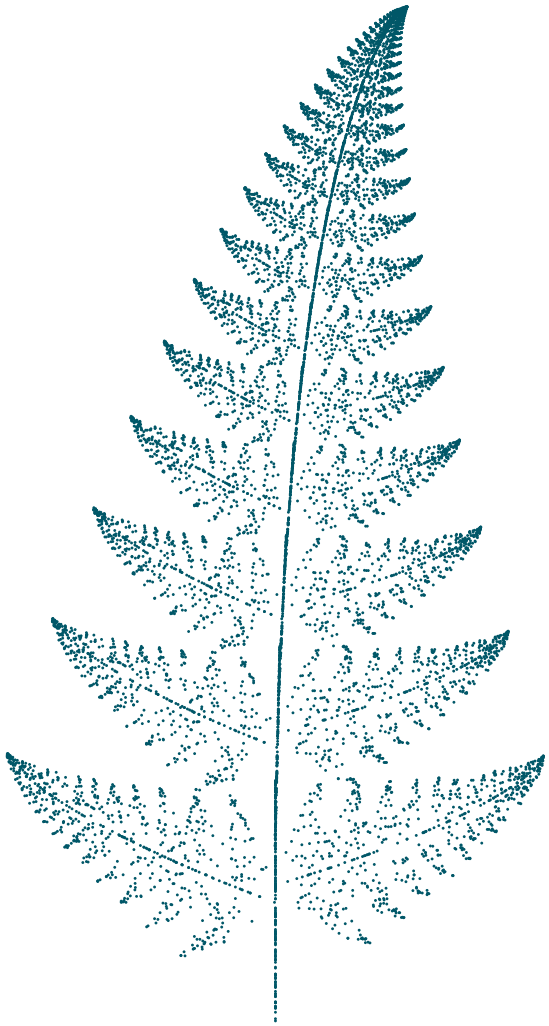
1. Traditional way: plot each point as it is calculated
2. Store each point in matrix, then plot the matrix
3. Preallocate the size of the matrix

Fern Fractals and Code Improvement

Fern 1 with n=10000 iterations
took 5.508156 seconds



Fern 2 with n=10000 iterations
took 1.305761 seconds



Fern 3 with n=10000 iterations
took 0.034057 seconds

Complex Numbers

- Introduce complex numbers

Complex Numbers

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- Addition and subtraction

Complex Numbers

- Introduce complex numbers
- Addition and subtraction
- Multiplication and de Moivre's formula

Chaos Game

- Generate vertices of the polygon as $\sin\theta + i\cos\theta$

Chaos Game

- Generate vertices of the polygon as $\sin\theta + i\cos\theta$
- Draw polygon.

Chaos Game

- Generate vertices of the polygon as $\sin\theta + i\cos\theta$
- Draw polygon.
- Draw vertices using different colors and markers.

Chaos Game

- Generate vertices of the polygon as $\sin\theta + i\cos\theta$
- Draw polygon.
- Draw vertices using different colors and markers.
- Preallocate the vector.

Chaos Game

- Generate vertices of the polygon as $\sin\theta + i\cos\theta$
- Draw polygon.
- Draw vertices using different colors and markers.
- Preallocate the vector.
- Generate seed as a random complex number.

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- Play the game for n turns:

Chaos Game

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- Play the game for n turns:
  - Simulate a roll of the die.

Chaos Game

- Generate vertices of the polygon as $\sin\theta + i\cos\theta$
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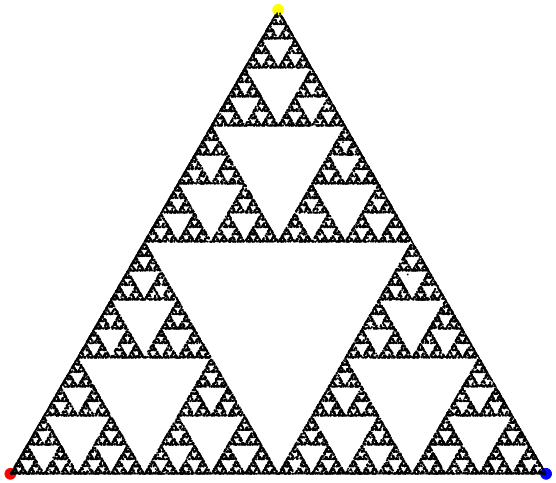
  - Simulate a roll of the die.
  - Next element is X of the midpoint between current element and vertex chosen.

Chaos Game

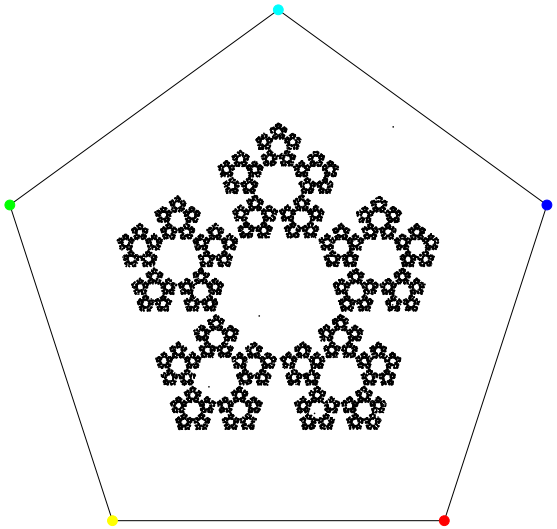
- Generate vertices of the polygon as $\sin\theta + i\cos\theta$
- Draw polygon.
- Draw vertices using different colors and markers.
- Preallocate the vector.
- Generate seed as a random complex number.
- Play the game for n turns:
  - Simulate a roll of the die.
  - Next element is X of the midpoint between current element and vertex chosen.
- Plot points.

Chaos Game

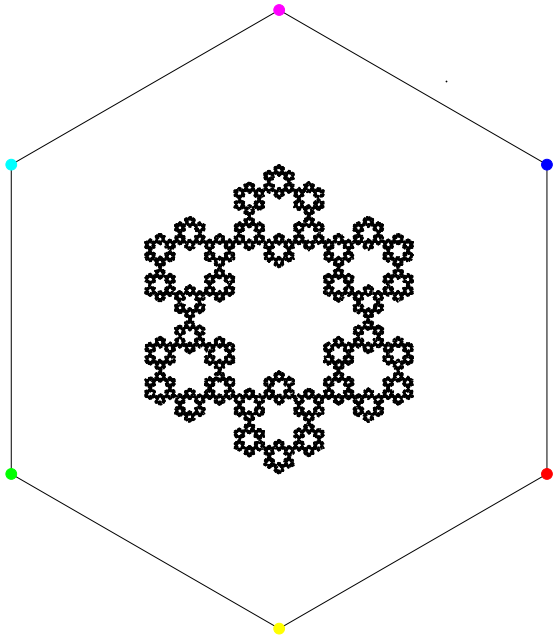
$$n = 30000$$



(a) $X = 1/2$

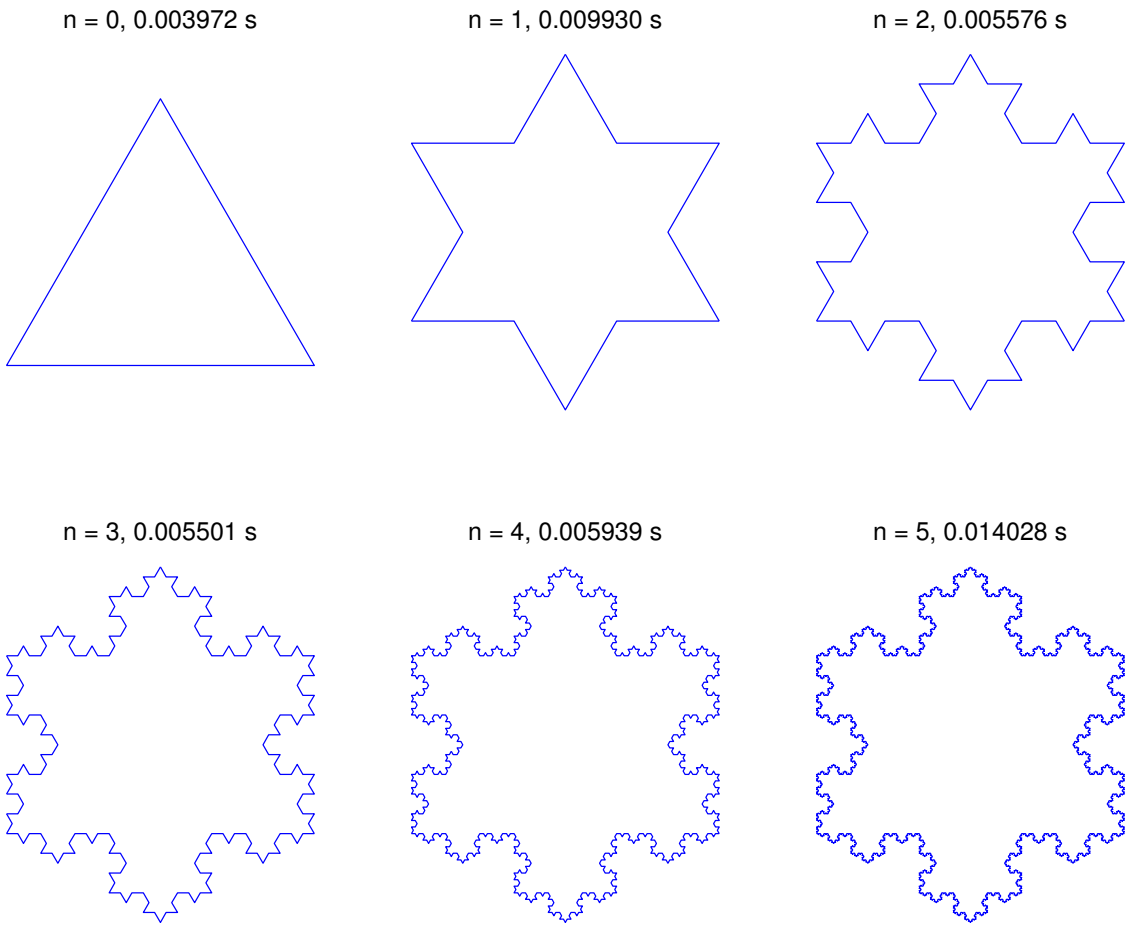


(b) $X = 3/8$



(c) $X = 1/3$

Line Replacement Fractals



Line Replacement Fractals

Given n-th iteration

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- Given pseudocode

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- Figure the geometry for the points

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Given n-th iteration

- Given pseudocode
- Figure the geometry for the points
- Plot the fractal

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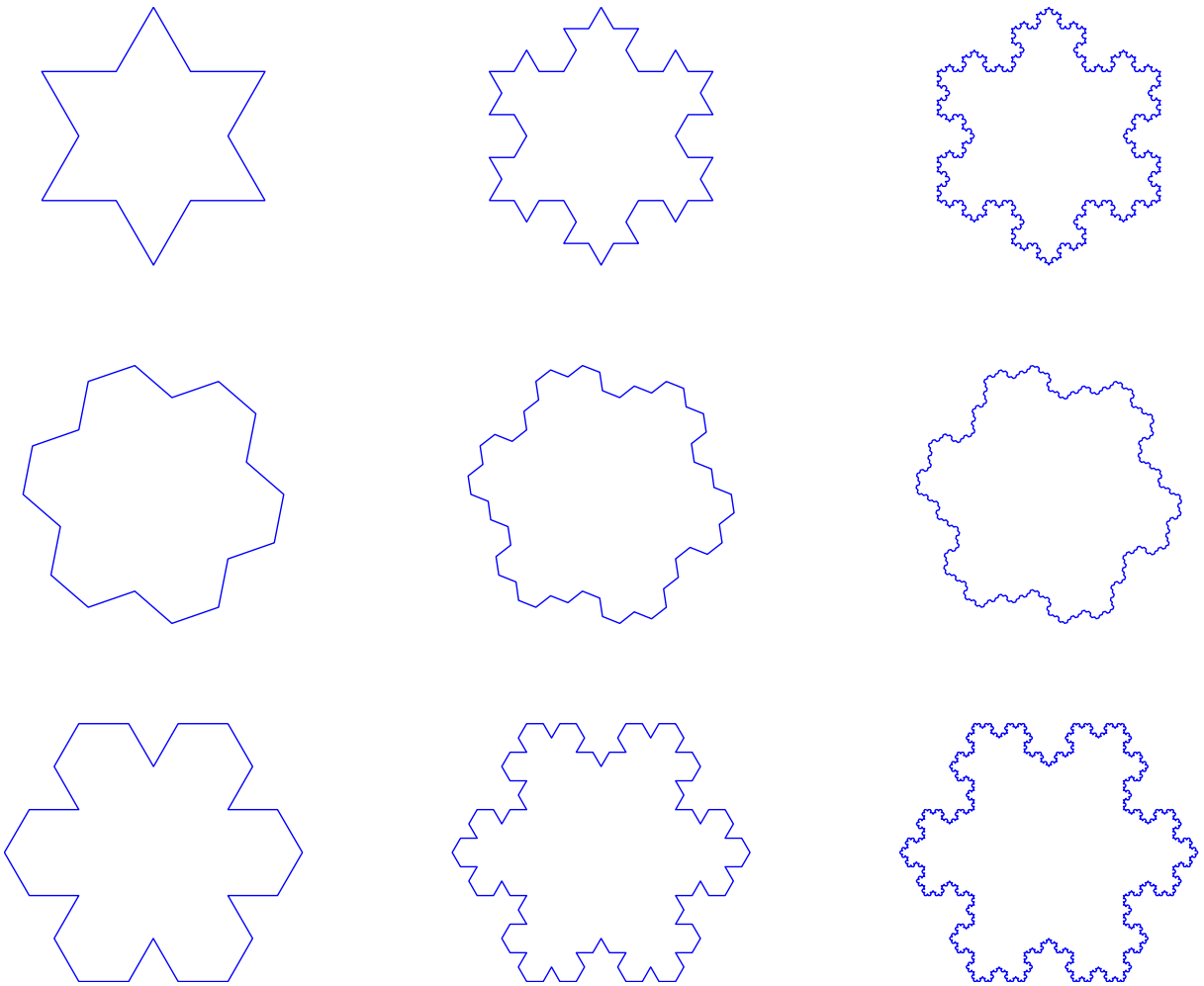
- Given pseudocode
- Figure the geometry for the points
- Plot the fractal
- Calculate the enclosed area and perimeter for small n

Line Replacement Fractals

Given n-th iteration

- Given pseudocode
- Figure the geometry for the points
- Plot the fractal
- Calculate the enclosed area and perimeter for small n
- Use geometric series for area and perimeter as $n \to \infty$

Examples



Future

Assignments could be expanded

Done in lower level courses

Explore other fractals

Thank you!

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