Use algebra and/or identities, get the equations in form of a **basic trig equation** like \( \cos x = a \), \( \sin x = b \), \( \tan x = c \), etc. where \( a, b \in [-1, 1] \) and \( c \in \mathbb{R} \).

**USEFUL FACTS** to help solve these:

1. \( \cos x \) and \( \sin x \) are **periodic** with period \( 2\pi \).
2. \( \tan x \) is periodic with period \( \pi \).
3. \( \cos(-x) = \cos x \) for any \( x \).
4. \( \sin(\pi - x) = \sin x \) for any \( x \).
5. \( \mathbb{Z} \) is short hand for the set of integers: \( \mathbb{Z} = \{0, \pm1, \pm2, \pm3, \ldots \} \).

To see the last two facts of the above list, draw the angles \( \frac{\pi}{6} \), \( -\frac{\pi}{6} \), \( \pi - \frac{\pi}{6} = \frac{5\pi}{6} \) and \( \pi - (-\frac{\pi}{6}) = \frac{7\pi}{6} \) and see how the sine, cosine and tangents are related. This helps me remember the facts 3 and 4 in order to use them.

There are always 2 solutions to a basic trig equation in one rotation of the unit circle. All other solutions are **coterminal** to these 2 solutions. For example, for \( \cos x = 0 \) has the solution \( x = \frac{\pi}{2} \) and \( \frac{3\pi}{2} \) and all others are coterminal, so can be gotten by adding/subtracting multiples of \( 2\pi \) to these. For the solution to \( \sin x = 0 \), there is \( x = 0 \) and \( x = \pi \). Many times we want solutions to be between 0 and \( 2\pi \), so in this case (and only this case), there would be 3: 0, \( \pi \) and \( 2\pi \) since \( 2\pi \) is coterminal to the angle 0.

**Solving \( \cos x = a \):**

If \( a = 0, \pm \frac{1}{2}, \pm \frac{\sqrt{2}}{2} \) (or \( \pm \frac{1}{\sqrt{2}} \)), \( \pm \frac{\sqrt{3}}{2} \) or \( \pm 1 \), then \( x \) is one of our special angles.

If not, then \( x = \cos^{-1} a \) is one of our solutions, call it \( s_1 \). \( s_1 = \cos^{-1} a \) is in exact form (for \( a \) not one of the special values listed above). Use your calculator to get a decimal approximation. Remember, by definition of inverse cosine, this angle \( s_1 \) is between 0 and \( \pi \) (inclusive). The other solution, by using fact 3, would be \( -s_1 \). All others would be coterminal. Many times you’re asked to give all solutions (which is implied if not stated otherwise). If so, you’d write:

\[
(\text{all solutions to } \cos x = a \text{ is needed}): \ s_1 + 2n\pi, \ -s_1 + 2n\pi, \ n \in \mathbb{Z}
\]

Other times you’re asked to state all solutions within a certain range, like all solutions in \([0, 2\pi]\). Since \( s_2 = -s_1 \) is not in the this range, you need to find the coterminal angle by adding \( 2\pi \). Thus you’d write:

\[
(\text{only solutions in } [0, 2\pi] \text{ needed}): \ s_1, \ -s_1 + 2\pi
\]

**Example:** \( \cos x = -\frac{1}{3} \). State both all solutions and solutions in \([0, 2\pi]\). Give your answers to 4 decimal places.

One solution is \( \cos^{-1} \left(-\frac{1}{3}\right) \approx 1.9106 \). The other solution would be \(-1.9106\). So all solutions would be \( \approx 1.9106 + 2n\pi, -1.9106 + 2n\pi, n \in \mathbb{Z} \) and the solutions between 0 and \( 2\pi \) would be \( \approx 1.9106 \) and \(-1.9106 + 2\pi \approx 4.3726 \).
Solving $\sin x = b$:

Like in the cosine case above, if $b = 0$, $\pm \frac{1}{2}$, $\pm \frac{\sqrt{2}}{2}$ (or $\pm \frac{1}{\sqrt{2}}$), $\pm \frac{\sqrt{3}}{2}$ or $\pm 1$, then $x$ is one of our special angles.

If not, then $x = \sin^{-1} b$ is one of our solutions, call it $s_1$. $s_1 = \sin^{-1} b$ is in exact form (for $b$ not one of the special values listed above). Use your calculator to get a decimal approximation. Remember, by definition of inverse sine, this angle $s_1$ is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (inclusive). The other solution, using fact 4 above, would be $s_2 = \pi - s_1$. All other solutions would be coterminal. Like in the cosine case, we may need to adjust these solutions in order for them to be between 0 and $2\pi$, if that is required.

Example: $\sin x = -\frac{1}{3}$. State both all solutions and solutions in $[0, 2\pi]$. Give your answers to 4 decimal places.

One solution is $\sin^{-1} \left(-\frac{1}{3}\right) \approx -0.3398$. Another solution is $\pi - (-0.3398) \approx 3.4814$. So all solutions would be $\approx -0.3398 + 2n\pi$, $3.4814 + 2n\pi$, $n \in \mathbb{Z}$ and solutions between 0 and $2\pi$ would be $\approx -0.3398 + 2\pi \approx 5.9434$ and $3.4814$.

Solving $\tan x = c$:

If $c = 0$, $\pm \frac{1}{\sqrt{3}}$ (or $\pm \frac{\sqrt{3}}{3}$), $\pm \sqrt{3}$, or $\pm 1$, then $x$ is one of our special angles.

If not, then $x = \tan^{-1} c$ is one of our solutions, call it $s_1$. $s_1 = \tan^{-1} c$ is in exact form (for $c$ not one of the special values listed above). Use your calculator to get a decimal approximation. Remember, by definition of inverse tangent, this angle $s_1$ is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (exclusive). Since $\tan x$ has period $\pi$, all other solutions are found by adding or subtracting multiples of $\pi$ to $s_1$. Thus you can quickly state all solutions or can find the solutions between 0 and $2\pi$ by adding multiples of $\pi$ to $s_1$.

Example: $\tan x = -\frac{1}{3}$. State both all solutions and solutions in $[0, 2\pi]$. Give your answers to 4 decimal places.

One solution is $\tan^{-1} \left(-\frac{1}{3}\right) \approx -0.3218$. The other solutions would be integer multiples of $\pi$. So all solutions would be and the solutions between 0 and $2\pi$ would be $\approx -0.3181 + \pi \approx 2.8198$ and $-0.3181 + 2\pi \approx 5.9614$.  
