Use algebra and/or identities, get the equations in form of a **basic trig equation** like $\cos x = a$, $\sin x = b$, $\tan x = c$, etc. where a, $b \in [-1, 1]$ and $c \in \mathbb{R}$.

USEFUL FACTS to help solve these:

1. $\cos x$ and $\sin x$ are **periodic** with period 2π.
2. $\tan x$ is periodic with period π.
3. $\cos(-x) = \cos x$ for any x.
4. $\sin(\pi - x) = \sin x$ for any x.
5. $\mathbb{Z}$ is short hand for the set of integers: $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$.

To see the last two facts of the above list, draw the angles $\frac{\pi}{6}$, $-\frac{\pi}{6}$, $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$ and $\pi - \left(-\frac{\pi}{6}\right) = \frac{7\pi}{6}$ and see how the sine, cosine and tangents are related. This helps me remember the facts 3 and 4 in order to use them.

There are always 2 solutions to a basic trig equation in one rotation of the unit circle. All other solutions are **coterminal** to these 2 solutions. For example, for $\cos x = 0$ has the solution $x = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ and all others are coterminal, so can be gotten by adding/subtracting multiples of 2π to these. For the solution to $\sin x = 0$, there is $x = 0$ and $x = \pi$. Many times we want solutions to be between 0 and 2π, so in this case (and only this case), there would be 3: 0, π and 2π since 2π is coterminal to the angle 0.

Solving $\cos x = a$**:**
If $a = 0$, $\pm\frac{1}{2}$, $\pm\frac{\sqrt{2}}{2}$ (or $\pm\frac{1}{\sqrt{2}}$), $\pm\frac{\sqrt{3}}{2}$ or ± 1, then x is one of our special angles.
If not, then $x = \cos^{-1} a$ is one of our solutions, call it s_1. $s_1 = \cos^{-1} a$ is in exact form (for a not one of the special values listed above). Use your calculator to get a decimal approximation. Remember, by definition of inverse cosine, this angle s_1 is between 0 and π (inclusive). The other solution, by using fact 3, would be $-s_1$. All others would be coterminal. Many times you're asked to give all solutions (which is implied if not stated otherwise). If so, you'd write:

$$\text{(all solutions to } \cos x = a \text{ is needed): } s_1 + 2n\pi,\ -s_1 + 2n\pi, n \in \mathbb{Z}$$

Other times you're asked to state all solutions within a certain range, like all solutions in $[0, 2\pi]$. Since $s_2 = -s_1$ is not in the this range, you need to find the coterminal angle by adding 2π. Thus you'd write:

$$\text{(only solutions in } [0, 2\pi] \text{ needed): } s_1, -s_1 + 2\pi$$

Example: $\cos x = -\frac{1}{3}$. State both all solutions and solutions in $[0, 2\pi]$. Give your answers to 4 decimal places.

One solution is $\cos^{-1}\left(-\frac{1}{3}\right) \approx 1.9106$. The other solution would be -1.9106. So all solutions would be $\approx 1.9106 + 2n\pi$, $-1.9106 + 2n\pi$, $n \in \mathbb{Z}$ and the solutions between 0 and 2π would be ≈ 1.9106 and $-1.9106 + 2\pi \approx 4.3726$

Solving $\sin x = b$**:**
Like in the cosine case above, if $b = 0$, $\pm\frac{1}{2}$, $\pm\frac{\sqrt{2}}{2}$ (or $\pm\frac{1}{\sqrt{2}}$), $\pm\frac{\sqrt{3}}{2}$ or ± 1, then x is one of our special angles.
If not, then $x = \sin^{-1} b$ is one of our solutions, call it s_1. $s_1 = \sin^{-1} b$ is in exact form (for b not one of the special values listed above). Use your calculator to get a decimal approximation. Remember, by definition of inverse sine, this angle s_1 is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (inclusive). The other solution, using fact 4 above, would be $s_2 = \pi - s_1$. All other solutions would be coterminal. Like in the cosine case, we may need to adjust these solutions in order for them to be between 0 and 2π, if that is required.

Example: $\sin x = -\frac{1}{3}$. State both all solutions and solutions in $[0, 2\pi]$. Give your answers to 4 decimal places.

One solution is $\sin^{-1}\left(-\frac{1}{3}\right) \approx -.3398$ Another solution is $\pi - (-.3398) \approx 3.4814$. So all solutions would be $\approx -.3398 + 2n\pi,\ 3.4814 + 2n\pi,\ n \in \mathbb{Z}$. and solutions between 0 and 2π would be $\approx -.3398 + 2\pi \approx 5.9434$ and 3.4814.

Solving $\tan x = c$**:**
If $c = 0$, $\pm\frac{1}{\sqrt{3}}$ (or $\pm\frac{\sqrt{3}}{3}$), $\pm\sqrt{3}$, or ± 1, then x is one of our special angles.
If not, then $x = \tan^{-1} c$ is one of our solutions, call it s_1. $s_1 = \tan^{-1} c$ is in exact form (for c not one of the special values listed above). Use your calculator to get a decimal approximation. Remember, by definition of inverse tangent, this angle s_1 is between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (exclusive). Since $\tan x$ has period π, all other solutions are found by adding or subtracting multiples of π to s_1. Thus you can quickly state all solutions or can find the solutions between 0 and 2π by adding multiples of π to s_1.

Example: $\tan x = -\frac{1}{3}$. State both all solutions and solutions in $[0, 2\pi]$. Give your answers to 4 decimal places.

One solution is $\tan^{-1}\left(-\frac{1}{3}\right) \approx -.3218$. The other solutions would be integer multiples of π. So all solutions would be and the solutions between 0 and 2π would be $\approx -.3181 + \pi \approx 2.8198$ and $-.3181 + 2\pi \approx 5.9614$.