Matlab Programming
Area Approximation [1] [2]

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[1] *Matlab, An Introduction with Applications, 2nd ed.* by Amos Gilat
[2] *Matlab Guide, 2nd ed.* by D. J. Higham and N. J. Higham

Approximating Integrals

Simplified definition:

$$\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{k=1}^{n} A_k$$

where

$$A_k\text{: area of approximating shapes (rectangles, etc)}$$

Split up the interval $[a, b]$ into n-subintervals:

$$a = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = b$$

They can be of equal width or unequal width

Using Rectangles

If looking at n rectangles of equal width, area of k-th rectangle is

$$A_k = f(x_k^*)\Delta x$$

where

$$\begin{aligned} \Delta x &= \text{width of each rectangle} \\ x_k^* &= \text{``sample point'' in } k\text{-th subinterval } x_k^* \in [x_{k-1}, x_k] \\ f(x_k^*) &= \text{height of rectangle} \end{aligned}$$

Left/Right Sums

Again we are using rectangles of equal width. The left sum is chosen as the left endpoint of each subinterval, while the right sum is chosen as the right endpoint of each subinterval.

$$\begin{aligned} n &= \text{number of rectangles} \\ \Delta x &= \frac{b-a}{n} \\ x_k &= a + k\Delta x, \quad k = 0, 1, \ldots n \\ \text{Left Sum: } x_k^* &= ? \\ \text{Right Sum: } x_k^* &= ? \end{aligned}$$

Midpoint and Trapezoid Rule

Midpoint Rule: x_k^* is the midpoint of each subinterval

$$x_k^* = ?$$

Trapezoid Rule: Area of trapezoid at the k-th interval

$$\Delta x \left(\frac{f(x_{k-1}) + f(x_k)}{2} \right)$$

Simpson's Rule

Use parabolas, not lines as top boundary of the region

$$\Delta x = \frac{b-a}{n}$$

Goal: Approximate $f(x)$ with a parabola. If $y_k = f(x_k)$, then

$$\int_a^b f(x)\ dx \approx \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 4y_{n-1} + y_n)$$

Note: n is assumed to be even