Math 302: Problem Set 6

Due: October 29, 2009 Notice: Due in 2 weeks

Email me your m-files and results as attachments in ONE email with each m-file having the format `firstname_lastname_prob.m`. Be sure to comment your code so I can follow it. The first problem is writing a function to do a specific task/calculation(s), while the second problem then uses your function to answer the questions for an application.

1. (50 points) Write an m-file, called `newton.m` that uses Newton's Method to find an approximate solution to the equation $f(x) = 0$, for a given function f. Your input should be:

   - the function f,
   - an initial guess x_0 for the solution,
   - the desired accuracy,
   - the maximum number of iterations allowed (so it will stop if accuracy cannot be reached).

   Note: the user must input the function as a symbolic function and then use the Symbolic Toolbox to calculate its derivative. The output should be the approximation to the solution of $f(x) = 0$. Your code should iterate until the absolute value of the difference between the last two iterations is less than the desired tolerance/accuracy OR the maximum number of iterations have been reached – in either case, an appropriate message should be printed on the screen (use `disp` or `fprintf`). The output of your function is the LAST iterate. Test your function for correctness by choosing to solve an equation of your choice (with a known solution).
   Grading: 10 points for correct header, 5 points for correct usage of symbolic variables, 5 points for correct differentiation, 10 points for accurate loop, 10 points for accurate Newton's Method, 10 points for commenting

2. (50 points) A car dealer sells a new car for $\$23,275$. She also offers to sell the same car for payments of \$475 per month for five years. What monthly interest rate is this dealer charging?
   To solve this problem, you will need to use the formula for the present value A of an annuity consisting of n equal payments of size R with an interest rate x per time period:

   $$A = \frac{R}{x}[1 - (1 + x)^{-n}].$$

   Create a script file called `newtonTrial.m` that solves the following

   (a) (10 points) For the above situation, get a polynomial of x (simplified as much as possible) that is set to 0 that we would need to solve to find the interest rate x. Put your solution as a comment.

   (b) (15 points) Graph the polynomial to find an interval that contains a positive root and a good initial guess for the interest rate. (Make a clear, good graph – maybe with `grid on`. State what your initial guess is in the title of your graph. Send a jpeg of this graph called `hw62b.jpg`.

(c) (15 points) Use Newton's method and the initial guess from part (b) to solve the equation to find the monthly interest rate.

(d) (10 points) What is the yearly interest rate (compounded monthly)? Hint: Think of a loan with principle P and interest rate x which is compounded monthly (and x is the monthly interest rate). After one year, the amount of the loan should be the same as if it was accounted for yearly. Take these, form an equation and from your x in the part c, calculate the yearly interest rate.

Note: The files `firstname_lastname_newton.m` and `firstname_lastname_newtonTrial.m` must be in the same directory. Be sure `firstname_lastname_newton.m` is called in your `firstname_lastname_newtonTrail.m`. Therefore, the user does not have to change the name of the file in order to run the code.