Math 421: Exam 2
Due: November 24, 2008

This is a closed book exam meaning no books, no notes, no calculators.
I understand and will uphold the ideals of academic
honesty as stated in the Honor Code.

____
Please Sign Name

____
Please Print Name

Start Time: __________
End Time: __________
Time Used: ___/240 min

Problem	Points	Score
1	15	
2	15	
3	15	
4	15	
5	15	
6	15	
7	10	
Total	100	

1. (15 points) Let f be a real-valued function with domain D in $\mathbb{R}$. Prove that f is continuous at x_0 if and only if for every monotone sequence (x_n) in D such that $x_n \to x$ we have $f(x_n) \to f(x)$.
   (Hint: Use the fact that every sequence has a monotone subsequence)

2. (15 points) Show that if (p_n) and (q_n) are Cauchy sequences then $|p_n - q_n|$ is convergent.

3. (15 points) Consider the sequence $\{a_n\} = \frac{1}{n^3+2}$. Find the N chosen to show $a_n \to 0$.

4. (15 points) Show $\lim_{x\to 1} x^2 + 4x + 1 = 6$.

5. (15 points) Let $f : [a, b] \to \mathbb{R}$ be a continuous function such that $f(x) > 0$ for all $x \in [a, b]$. Prove that there exists a number $\alpha > 0$ such that $f(x) \geq \alpha$ for all $x \in [a, b]$.

6. (15 points) Suppose that $f : [a, b] \to \mathbb{R}$ is continuous and that $f([a, b]) \subseteq \mathbb{Q}$. Prove that f is constant on $[a, b]$.

7. (10 points) Prove or give a counterexample: If $f : A \to B$ is uniformly continuous on A and $g : B \to C$ is uniformly continuous on B, then $g \circ f : A \to C$ is uniformly continuous on A.

Mid-semester Course Evaluation

Please tear off and submit separately

1. How is the pace of the course?

2. How is the teaching style of the course?

3. How is the presentation of the course material?

4. Is the course stimulating your intellectual curiosity?

5. Explain aspects of the course that you like.

6. Explain aspects of the course that you would like changed.

7. Questions/Comments/Vent