Math 421: Take - Home Final
Due: December 12, 2008

This is an open book, open notes, but close friends exam
meaning you are not to consult with anyone other than the professor for the exam.
I understand and will uphold the ideals of academic
honesty as stated in the Honor Code.

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Please Sign Name

_____

Please Print Name

Start Time: _____
End Time: _____
Time Used: ___/240 min

Problem	Points	Score
1	15	
2	10	
3	10	
4	15	
5	10	
6	10	
7	15	
8	15	
Total	100	

1. (15 points) Let S be the Cartesian coordinate plane $\mathbb{R} \times \mathbb{R}$ and define a relation $\mathbf{R}$ on S by $(a, b)\mathbf{R}(c, d)$ iff $a + d = b + c$. Verify that $\mathbf{R}$ is an equivalence relation.

2. (10 points) Let S be the set of all circles in the plane. Define $f : S \to [0, \infty)$ by $f(C) =$ the area of C, for all $C \in S$ Is f injective? Is f surjective?

3. (10 points) Prove that an accumulation point of a set S is either an interior point or a boundary point of S.

4. (15 points) Consider the sequence $\{a_n\} = \{\frac{1}{3n^4+5}\}$. Find the N chosen to show $a_n \to 0$. In other words, prove using the definition that $a_n \to 0$.

5. (10 points) Using definitions, prove $x^2 - 3x + 1$ is continuous at 2.

6. (10 points) For which value of x is $f(x) = \frac{2}{x-1}$ not differentiable and why (using definitions)?

7. (15 points) Let f be differentiable on $\mathbb{R}$. Suppose that $f(0) = 0$ and that $1 \leq f'(x) \leq 2$ for all $x \geq 0$. Prove that $x \leq f(x) \leq 2x$ for all $x \geq 0$.

8. (15 points) Suppose that $f(x) = 2x$ for all $x \in [0, b]$. Show that f is integrable and that $\int_0^b f(x)\,dx = b^2$.