Homework 1: Linear Equations

Due: Feb 1, 2012

1. Assume $A \in \mathbf{C}^{m \times m}$ is nonsingular. Show that if A has an LU factorization then for each k with $1 \leq k \leq m$ the upper-left $k \times k$ block of A is nonsingular.

2. Show that L_k and L'_k obtained at the k-th step of Gaussian Elimination have the same structure. Here
$$L'_k = P_{m-1} \dots P_{k+1} L_k P_{k+1}^{-1} \dots P_{m-1}^{-1}.$$

3. Calculate by hand the LU decomposition of
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}$$
both with and without partial-pivoting. Then use both techniques to solve
$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 11 \end{pmatrix}$$

4. Moler 2.7, 2.8, 2.20, 2.21