

## Homework 2: Least Squares

Due: Feb 15, 2012

1. Let  $P$  be a projector. Prove that the

$$\text{Null}(I - P) = \text{Range}(P)$$

2. Create three programs to compute the QR factorization using

- classical Gram-Schmidt,
- modified Gram-Schmidt,
- Householder transformations

Test your three methods on the Vandermonde matrix that can be created in Matlab using the code:

```
m=100;n=15;
t=linspace(0,1,m)';
% Create Vandermonde
A = [];
for i = 1:n
    A = [A t.^(i-1)];
end
```

Comment on the orthogonality of your 3 corresponding Q matrices. Then test the method with the right-hand side being created by the following code:

```
x = ones(n,1);
b = A*x;
```

Which method would you use in practice and why?

3. Let  $A$  be an  $m \times n$  matrix ( $m \geq n$ ), and let  $A = \hat{Q}\hat{R}$  be a reduced QR factorization. Show that  $A$  has rank  $n$  if and only if all the diagonal entries of  $\hat{R}$  are nonzero.
4. Moler 5.4, 5.5