

Homework 3: Eigenvalues and SVD

Due: Feb 24, 2012

1. Prove similar matrices have the same eigenvalues. Matrices A and B are similar if there exists a matrix P such that $B = P^{-1}AP$. Will similar matrices have the same eigenvectors?
2. Let A be a symmetric matrix. Prove that if x and y are eigenvectors corresponding to distinct eigenvalues then x and y are orthogonal.
3. Prove that $A \in \mathbf{R}^{m \times n}$ with $m \geq n$ is full rank if and only if the singular values of A are nonzero if and only if $A^T A$ is nonsingular.
4. Given $A \in \mathbf{R}^{m \times n}$, prove $\|A\|_2 = \sigma_1$ the largest singular value of A . Recall $\|A\|_2 = \max_{\|x\|=1} \|Ax\|_2$.
5. MATLAB: For the programming exercise of this problem set, you will be looking at compression of pictures. First, let's experiment with a picture stored in MATLAB. To begin, let's open the image called `rice.png`.


```
close all
A=imread('rice.png');
A = A(:,:,1);
A=im2double(A);
imshow(A)
```

 - (a) Now experiment with MATLAB's built in SVD operation to calculate a low rank approximation. Print a 1-rank, 20-rank, 50-rank, and 100-rank approximation and comment on the savings for using such a low rank approximation.
 - (b) Load your favorite picture into MATLAB and experiment with different low rank approximations. Calculate which low rank approximation gives the best approximation while saving the most space. In other words, which rank gives you the most bang for the buck! Be sure to print your picture and your low rank approximation and give an explanation on why you chose the low rank approximation.