Homework 6: Quadrature
Due: April 18, 2012

1. Basic Quadrature rules. The Error Function erf(x) is defined by the definite integral

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

Given $\text{erf}(1) = 0.84270079294971$, calculate approximate values $Q \approx \text{erf}(1)$ and relative errors (i.e. $|\text{erf}(1) - Q|/\text{erf}(1)$) for each of the following methods (just write down the formulas and your calculated answers)

   (a) Trapezoid Rule

   (b) Simpsons Rule

   (c) (Time Permitting) 3-point Recursion Gaussian Quadrature (hint: use undetermined coefficients given the zeros of the Legendre Polynomial $P_3(x) = x^3 - \frac{3}{5}x$. Be careful about the limits of integration).

2. Consider quadrature rules for the integral $\int_a^b f(x)dx$ for fixed a and b, $a < b$.

   (a) Prove there is no quadrature rule of the form

$$I(f) = w_0 f(a) + w_1 f(b)$$

   that is exact for all quadratic polynomials.
   (Here w_0 and w_1 are constants that can depend on a and b, but not on the polynomial.)

   (b) Prove that Simpson's rule is exact for all cubic polynomials.

3. Update the Moler program `quadgui` to create a new adaptive recursive algorithm `quadguiMOD` that is based on the trapezoid rule and midpoint rule. Test on a few functions and compare the algorithms. Which method would you use and why?