Math 427: Problem Set 7

Due: April 30, 2012

1. Is the method
$$x_{i+1} = -\frac{3}{2}x_i + 3x_{i-1} - \frac{1}{2}x_{i-2} + 3hf(t_i, x_i)$$
consistent? Is the method order - p accurate? If so, for what p?

2. Ed Lorenz was an American mathematician and meteorologist, and a pioneer of chaos theory. He was interested in weather modeling and came up with a simple differential equation system:
$$\begin{aligned} \frac{dx}{dt} &= \sigma(y - x) \\ \frac{dy}{dt} &= \rho x - y - xz \\ \frac{dz}{dt} &= -\beta z + xy \end{aligned}$$
He chose $\sigma = 10$, $\beta = 8/3$ and $\rho = 28$. Apply a numerical ODE technique to solve this system with initial conditions $[0, 1, 0]$ and $[0, 1.001, 0]$ for time $t = [0, 35]$. Notice how this slight change cause differences in your solution by plotting:

   - 3D graph x, y, z
   - 2D graph x verses t and y verses t

   Note that this is a classic example of chaos – how a small change can cause drastic change in the future.

3. MATLAB: Moler 7.16