Homework 1: Gaussian Elimination
Due: January 31, 2014

1. Show that L_k and L'_k obtained at the k-th step of Gaussian Elimination have the same structure. Here

$$L'_k = P_{m-1} \ldots P_{k+1} L_k P_{k+1}^{-1} \ldots P_{m-1}^{-1}.$$

2. Calculate by hand the LU decomposition of

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}$$

   both with and without partial-pivoting. Then use both techniques to solve by hand

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 11 \end{pmatrix}$$

3. G&C: 8, 9, 10

4. MATLAB:

   - Generate a function `GaussPP.m` that takes as input an $n \times n$ matrix $\mathbf{A}$ and returns the matrices $\mathbf{P}, \mathbf{L}, \mathbf{U}$ such that $\mathbf{PA} = \mathbf{LU}$.

   - Create a driver that uses `GaussPP.m` to solve

$$\begin{pmatrix} 2 & 1 & 5 & 1 \\ 3 & 2 & 11 & 1 \\ 2 & 8 & 7 & 3 \\ 7 & 4 & 4 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

   Please note the solution $\mathbf{x}$ and the matrices $\mathbf{P}, \mathbf{L}$, and $\mathbf{U}$.