Homework 6: Interpolation

Due: April 11, 2014

1. Consider quadrature rules for the integral $\int_a^b f(x)dx$ for fixed a and b, $a < b$.

   (a) Prove there is no quadrature rule of the form

   $$I(f) = w_0 f(a) + w_1 f(b)$$

   that is exact for all quadratic polynomials.
   (Here w_0 and w_1 are constants that can depend on a and b, but not on the polynomial.)

   (b) Prove that Simpson's rule is exact for all cubic polynomials.

2. (a) Suppose we wish to approximate the integral $\int_{-1}^{1} f(x)\ dx$ with the quadrature rule

   $$\int_{-1}^{1} f(x)\ dx \approx Af(0) + Bf'(0).$$

   How should A and B be selected to make this rule exact for all linear polynomials?

   (b) Suppose we improve this rule to

   $$\int_{-1}^{1} f(x)\ dx \approx Af(0) + Bf'(0) + Cf''(0).$$

   How should A, B, and C be selected to make this rule exact for all quadratic polynomials?

   (c) With three evaluations of f, Simpson's rule is exact for all cubic polynomials. Is the same true of your method in (b)?

   (d) Assuming f is sufficiently differentiable, use the Taylor expansion with remainder term to derive a bound on

   $$\left|\int_{-1}^{1} f(x)\ dx - \Big(Af(0) + Bf'(0) + Cf''(0)\Big)\right|.$$

   (e) Why is the method you derived in (b) less famous than Simpson's rule?

3. G&C: 10.6