Six Degree of Freedom Point Correspondences

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***Abstract*—In this paper we develop the best homogeneous matrix transformation to fit two streams of dynamic six degree of freedom (6DOF) data. In particular, we compare object position and orientation results from two 6DOF sources. A problem that arises when comparing these two data streams is that they are not necessarily in the same coordinate system. Therefore, a method to transform one coordinate system to the other is needed. We solve this problem by developing an optimization problem that minimizes the space between each coordinate system. In other words, we construct a rotation and translation which best transforms one coordinate space to the other.**

I. Introduction

With the advent of newer and more technologically advanced robotic vision systems, there is greater need for novel mathematical techniques to calibrate these systems. In particular, there is a need to calibrate six degrees of freedom (6DOF) sensors which track not only the position (3DOF) but also the orientation of an object. These six degrees of freedom represent translations along three perpendicular axes: left and right (along the x-axis), forward and backward (along the y-axis), and up and down (along the z-axis); along with the rotations about those three perpendicular axes ($\mathbf{R}_x$, $\mathbf{R}_y$, and $\mathbf{R}_z$). They may be arranged as a homogeneous transformation

$$\mathbf{H} = \begin{pmatrix} \mathbf{R} & \mathbf{t} \\ 0 & 1 \end{pmatrix},$$

where $\mathbf{R} = \mathbf{R}_x\mathbf{R}_y\mathbf{R}_z$ represents the orientation of a given object and $\mathbf{t} = (x, y, z)^T$ represents the position of the given object. Given two streams of such six degrees of freedom data,

$$\begin{aligned} \mathbf{X} &= \left[\begin{pmatrix} R_0 & t_0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} R_1 & t_1 \\ 0 & 1 \end{pmatrix}, \ldots, \begin{pmatrix} R_{n-1} & t_{n-1} \\ 0 & 1 \end{pmatrix} \right] \\ \widehat{\mathbf{X}} &= \left[\begin{pmatrix} \widehat{R}_0 & \widehat{t}_0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \widehat{R}_1 & \widehat{t}_1 \\ 0 & 1 \end{pmatrix}, \ldots, \begin{pmatrix} \widehat{R}_{n-1} & \widehat{t}_{n-1} \\ 0 & 1 \end{pmatrix} \right], \end{aligned}$$

this paper constructs the best rotation $\mathbf{R}$ and translation $\mathbf{t}$ that fits the data. In other words, the best homogeneous matrix $\mathbf{H} = \begin{pmatrix} \mathbf{R} & \mathbf{t} \\ 0 & 1 \end{pmatrix}$ that minimizes

$$\min_{\mathbf{H}} \|\mathbf{H}\mathbf{X} - \widehat{\mathbf{X}}\|^2 \tag{1}$$

is constructed.

The solution $\mathbf{H}$ to minimization problem (1) involves a two step process:

1) Find the rotation $\mathbf{R}$ that minimizes

$$\min_{\mathbf{R}} \left\| \mathbf{R} \begin{pmatrix} R_0 & \mathbf{t}_0 & \ldots & R_{n-1} & \mathbf{t}_{n-1} \end{pmatrix} - \begin{pmatrix} \widehat{R}_0 & \widehat{\mathbf{t}}_0 & \ldots & \widehat{R}_{n-1} & \widehat{\mathbf{t}}_{n-1} \end{pmatrix} \right\|^2 \tag{2}$$

where

$$\mathbf{t}_i = t_i - t \quad \text{and} \quad t = \frac{1}{n}\sum_{i=0}^{n-1} t_i$$

$$\widehat{\mathbf{t}}_i = \widehat{t}_i - \widehat{t} \quad \text{and} \quad \widehat{t} = \frac{1}{n}\sum_{i=0}^{n-1} \widehat{t}_i$$

2) Set the best transformation

$$\mathbf{t} = \widehat{t} - \mathbf{R}t, \tag{3}$$

where $\mathbf{R}$ is calculated from Step 1.

The simpler problem of finding a closed-form solution to the best rotation and translation to fit two sets of three-dimensional point correspondences (which represents only position and hence 3DOF) has been around since the 1980's [1], [2]. Most formulations are reduced to finding a rotation $\mathbf{R}$ and translation $\mathbf{t}$ that solves

$$\min_{\mathbf{R},\mathbf{t}} \|\hat{\mathbf{X}} - (\mathbf{R}\mathbf{X} + \mathbf{t})\|^2 \tag{4}$$

where $\mathbf{X}$ and $\hat{\mathbf{X}}$ are $3 \times n$ matrices such that the i-th column of $\hat{\mathbf{X}}$ is given by

$$\hat{\mathbf{X}}_i = \mathbf{R}\mathbf{X}_i + \mathbf{t} + \mathbf{E}_i,$$

where $\mathbf{X}_i$ is the i-th column of $\mathbf{X}$ and $\mathbf{E}_i$ is a noise vector. This problem is commonly known as the *absolute orientation* problem. One of the issues with this problem is that there are certain cases where there are many – if not infinite – solutions to minimization problem (4) [1]. An example of this case is when all the points lie on the same line. This degeneracy is not a problem with the 6DOF representation. In Section IV, an example will be presented which compares solutions calculated using the absolute orientation (3DOF) problem (4) with the homogeneous problem (6DOF) (1) introduced here.

Historically, there are four main approaches to finding closed form solutions of the absolute orientation problem. The first method by Arun, Huang, and Blostein [1] is based on finding the best orthogonal matrix which fits the set of data and declares that the best rotation. An equivalent method - by Horn, Hilden, and Negahdaripour [2] - looks for the square-root of a symmetric matrix to represent rotation. A problem with both of these methods is that the matrix that is calculated may not necessarily be a rotation (in fact it is a reflection). Therefore, the results from the algorithm may have to be disregarded. In contrast, the method that is presented here is guaranteed to be a rotation matrix. The last two approaches – one by Horn [3] and the other by Walker, Shao, and Volz [4] – are based on quaternions. Modern extensions of the conventional four methods have been formulated by Umeyama [5] and Kanatani [6].

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In this paper, $\|\cdot\|$ denotes the Frobenius norm, so

$$\|\mathbf{A}\| = \sqrt{\operatorname{tr}(\mathbf{A}\mathbf{A}^T)}$$

where ^{T} denotes the transpose operator. And, tr() denotes the matrix trace operation, while $\operatorname{diag}(d_1 \dots d_n)$ represents the diagonal matrix with entries $d_1 \dots d_n$.

II. Simplifying Rotation and Translation

Here, we will outline the methodology that reduces the original system (1) to the two-step process shown in Equation (2) and Equation (3). First, observe that

$$\begin{aligned}
&\|\mathbf{H}\mathbf{X} - \widehat{\mathbf{X}}\|^2 = \\
= \ & \left\| \begin{pmatrix} \mathbf{R} & \mathbf{t} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} R_0 & t_0 & \dots & R_{n-1} & t_{n-1} \\ 0 & 1 & \dots & 0 & 1 \end{pmatrix} - \begin{pmatrix} \widehat{R}_0 & \widehat{t}_0 & \dots & \widehat{R}_{n-1} & \widehat{t}_{n-1} \\ 0 & 1 & \dots & 0 & 1 \end{pmatrix} \right\|^2 \\
= \ & \left\| \begin{pmatrix} \mathbf{R}R_0 - \widehat{R}_0 & \mathbf{R}t_0 + \mathbf{t} - \widehat{t}_0 & \dots & \mathbf{R}R_{n-1} - \widehat{R}_{n-1} & \mathbf{R}t_{n-1} + \mathbf{t} - \widehat{t}_{n-1} \\ 0 & 0 & \dots & 0 & 0 \end{pmatrix} \right\|^2 \\
= \ & \left\| \mathbf{R} \begin{pmatrix} R_0 & \dots & R_{n-1} \end{pmatrix} - \begin{pmatrix} \widehat{R}_0 & \dots & \widehat{R}_{n-1} \end{pmatrix} \right\|^2 + \sum_{i=0}^{n-1} \|\mathbf{R}t_i + \mathbf{t} - \widehat{t}_i\|^2 \quad (5)
\end{aligned}$$

Now, let the centroids for the two data sets be given by

$$t = \frac{1}{n}\sum_{i=0}^{n-1} t_i \quad \text{and} \quad \widehat{t} = \frac{1}{n}\sum_{i=0}^{n-1} \widehat{t}_i$$

and define

$$\mathbf{T} = \mathbf{t} + \mathbf{R}t - \widehat{t}.$$

Then for $i = 0, \dots, n-1$

$$\mathbf{t}_i = t_i - t \quad \text{and} \quad \widehat{\mathbf{t}}_i = \widehat{t}_i - \widehat{t}.$$

Therefore,

$$\begin{aligned}
&\sum_{i=0}^{n-1} \|\mathbf{R}t_i + \mathbf{t} - \widehat{t}_i\|^2 = \\
= \ & \sum_{i=0}^{n-1} \|\mathbf{R}(t_i - t) - (\widehat{t}_i - \widehat{t}) + \mathbf{t} + \mathbf{R}t - \widehat{t}\|^2 \\
= \ & \sum_{i=0}^{n-1} \|\mathbf{R}\mathbf{t}_i - \widehat{\mathbf{t}}_i + \mathbf{T}\|^2 \\
= \ & \sum_{i=0}^{n-1} \|\mathbf{R}\mathbf{t}_i - \widehat{\mathbf{t}}_i\|^2 + 2\mathbf{T}^T \left(\sum_{i+1}^{n} \mathbf{R}\mathbf{t}_i - \widehat{\mathbf{t}}_i \right) + n\|\mathbf{T}\|^2 \\
= \ & \sum_{i=0}^{n-1} \|\mathbf{R}\mathbf{t}_i - \widehat{\mathbf{t}}_i\|^2 + n\|\mathbf{T}\|^2
\end{aligned}$$

since

$$\sum_{i=0}^{n-1} \mathbf{R}\mathbf{t}_i - \widehat{\mathbf{t}}_i = 0.$$

So, if Equation (5) is minimized then

$$\begin{aligned}
&\min_{\mathbf{H}} \|\mathbf{H}\mathbf{X} - \widehat{\mathbf{X}}\|^2 \\
= \ & \min_{\mathbf{R},\mathbf{t}} \left\| \mathbf{R} \begin{pmatrix} R_0 & \dots & R_{n-1} \end{pmatrix} - \begin{pmatrix} \widehat{R}_0 & \dots & \widehat{R}_{n-1} \end{pmatrix} \right\|^2 \\
& + \sum_{i=0}^{n-1} \|\mathbf{R}t_i + \mathbf{t} - \widehat{t}_i\|^2 \\
= \ & \min_{\mathbf{R},\mathbf{t}} \left\| \mathbf{R} \begin{pmatrix} R_0 & \dots & R_{n-1} \end{pmatrix} - \begin{pmatrix} \widehat{R}_0 & \dots & \widehat{R}_{n-1} \end{pmatrix} \right\|^2 \\
& + \sum_{i=0}^{n-1} \|\mathbf{R}\mathbf{t}_i - \widehat{\mathbf{t}}_i\|^2 + n\|\mathbf{T}\|^2 \\
= \ & \min_{\mathbf{R},\mathbf{t}} \left\| \mathbf{R} \begin{pmatrix} R_0 & \mathbf{t}_0 & \dots & R_{n-1} & \mathbf{t}_{n-1} \end{pmatrix} - \begin{pmatrix} \widehat{R}_0 & \widehat{\mathbf{t}}_0 & \dots & \widehat{R}_{n-1} & \widehat{\mathbf{t}}_{n-1} \end{pmatrix} \right\|^2 \\
& + n\|\mathbf{T}\|^2
\end{aligned}$$

Note for any given rotation $\mathbf{R}$, we can set $\mathbf{T} = 0$ by allowing

$$\mathbf{t} = \widehat{t} - \mathbf{R}t \Rightarrow \mathbf{T} = \mathbf{t} + \mathbf{R}t - \widehat{t} = 0. \quad (6)$$

Thus, in order to calculate Equation (1), we first calculate $\mathbf{R}$ that minimizes

$$\min_{\mathbf{R}} \left\| \mathbf{R} \begin{pmatrix} R_0 & \mathbf{t}_0 & \dots & R_{n-1} & \mathbf{t}_{n-1} \end{pmatrix} - \begin{pmatrix} \widehat{R}_0 & \widehat{\mathbf{t}}_0 & \dots & \widehat{R}_{n-1} & \widehat{\mathbf{t}}_{n-1} \end{pmatrix} \right\|^2,$$

then we set

$$\mathbf{t} = \widehat{t} - \mathbf{R}t.$$

A. Finding $\mathbf{R}$

In the previous section, we found that finding the best fitting homogeneous transformation matrix is dependent on finding the best rotation that minimizes Equation (2):

$$\min_{\mathbf{R}} \left\| \mathbf{R} \begin{pmatrix} R_0 & \mathbf{t}_0 & \dots & R_{n-1} & \mathbf{t}_{n-1} \end{pmatrix} - \begin{pmatrix} \widehat{R}_0 & \widehat{\mathbf{t}}_0 & \dots & \widehat{R}_{n-1} & \widehat{\mathbf{t}}_{n-1} \end{pmatrix} \right\|^2.$$

For simplicity, this problem will be reformulated to

$$\min_{\mathbf{R}} \left\| \mathbf{R}\mathsf{X} - \widehat{\mathsf{X}} \right\|^2 \quad (7)$$

where

$$\begin{aligned}
\mathsf{X} &= \begin{pmatrix} R_0 & \mathbf{t}_0 & \dots & R_{n-1} & \mathbf{t}_{n-1} \end{pmatrix} \\
\widehat{\mathsf{X}} &= \begin{pmatrix} \widehat{R}_0 & \widehat{\mathbf{t}}_0 & \dots & \widehat{R}_{n-1} & \widehat{\mathbf{t}}_{n-1} \end{pmatrix}.
\end{aligned}$$

However,

$$\left\| \mathbf{R}\mathsf{X} - \widehat{\mathsf{X}} \right\|^2 = \left\| \mathsf{X} \right\|^2 - 2\operatorname{tr}(\mathbf{R}\mathsf{X}\widehat{\mathsf{X}}^T) + \left\| \widehat{\mathsf{X}} \right\|^2$$

Therefore, the $\mathbf{R}$ that solves the minimization problem (7) is equivalent to the rotation matrix $\mathbf{R}$ that solves

$$\max_{\mathbf{R}} \operatorname{tr}(\mathbf{R}\mathsf{X}\widehat{\mathsf{X}}^T) \quad (8)$$

There is a plethora of research on finding the best rotation matrix $\mathbf{R}$. Most of these methods are based on finding the best orthogonal matrix that fits the data. In most applications, this method works. However, there can be instances where the best orthogonal matrix that is produced could have determinant -1, meaning that the best orthogonal matrix is not a rotation but actually a reflection. In this section, we will describe a method for calculating the best rotational approximation to a set of data that is guaranteed to have determinant 1. This work is equivalent to Umeyama's work [5].

In order to construct the best rotation, the following Lemma will be of importance.

Lemma 2.1: For a given 3×3 matrix $\mathbf{M}$ and rotation $\mathbf{R}$

$$\text{tr}(\mathbf{RM}) \leq \text{tr}(\mathbf{D}\Sigma), \tag{9}$$

where

$$\mathbf{D} = \begin{cases} \text{diag}(1,1,1) & \text{if } \det(\mathbf{VU}^T) = 1, \\ \text{diag}(1,1,-1) & \text{if } \det(\mathbf{VU}^T) = -1 \end{cases}$$

and the full singular value decomposition (SVD) of

$$\mathbf{M} = \mathbf{U}\Sigma\mathbf{V}^T.$$

Proof: First notice that

$$\text{tr}(\mathbf{RM}) = \text{tr}(\mathbf{RU}\Sigma\mathbf{V}^T) = \text{tr}(\mathbf{R}(\mathbf{UDV}^T)\mathbf{D}\Sigma),$$

since $\mathbf{D}^2 = \mathbf{I}$ and $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA})$ for matrices $\mathbf{A}$ and $\mathbf{B}$ of appropriate degree. But $\widehat{\mathbf{R}} = \mathbf{R}(\mathbf{UDV}^T)$ is an orthogonal matrix with determinant 1 and hence a rotation matrix. Therefore,

$$\text{tr}(\mathbf{RM}) = \text{tr}(\widehat{\mathbf{R}}\mathbf{D}\Sigma) \leq \text{tr}(\mathbf{D}\Sigma)$$

■

Moreover, if a rotation $\mathbf{R}$ can be constructed such that

$$\text{tr}(\mathbf{R}\mathsf{X}\widehat{\mathsf{X}}^T) = \text{tr}(\mathbf{D}\Sigma),$$

then the minimization problem (7) is solved.

Theorem 2.2: The solution to the maximization problem (8) is

$$\mathbf{R} = \mathbf{VDU}^T$$

where the full SVD of the 3×3 matrix

$$\mathsf{X}\widehat{\mathsf{X}}^T = \mathbf{U}\Sigma\mathbf{V}^T$$

and

$$\mathbf{D} = \begin{cases} \text{diag}(1,1,1) & \text{if } \det(\mathbf{VU}^T) = 1, \\ \text{diag}(1,1,-1) & \text{if } \det(\mathbf{VU}^T) = -1 \end{cases}$$

Proof: From Lemma 2.1, the maximization problem (8) is solved if a rotation matrix $\mathbf{R}$ can be constructed such that

$$\text{tr}(\mathbf{R}\mathsf{X}\widehat{\mathsf{X}}^T) = \text{tr}(\mathbf{D}\Sigma).$$

Let

$$\mathbf{R} = \mathbf{VDU}^T.$$

Then

$$\text{tr}(\mathbf{R}\mathsf{X}\widehat{\mathsf{X}}^T) = \text{tr}([\mathbf{VDU}^T][\mathbf{U}\Sigma\mathbf{V}^T]) = \text{tr}(\mathbf{D}\Sigma).$$

■

Therefore, the optimal homogeneous matrix $\mathbf{H} = \left(\begin{smallmatrix} \mathbf{R} & \mathbf{t} \\ 0 & 1 \end{smallmatrix}\right)$ may be constructed by

1) Setting
$$\mathbf{R} = \mathbf{VDU}^T,$$
where the SVD of
$$\mathsf{X}\widehat{\mathsf{X}}^T = \mathbf{U}\Sigma\mathbf{V}^T$$
and
$$\mathbf{D} = \begin{cases} \text{diag}(1,1,1) & \text{if } \det(\mathbf{VU}^T) = 1, \\ \text{diag}(1,1,-1) & \text{if } \det(\mathbf{VU}^T) = -1 \end{cases}.$$

2) Setting
$$\mathbf{t} = \widehat{t} - \mathbf{R}t.$$

III. Error Metrics

For many applications, it is beneficial to understand how well the homogeneous matrix $\mathbf{H}$ fits the orientation of the 6DOF data independently of the position of the 6DOF data. Therefore, a description of separating the orientations from the positions in minimization problem (1), i.e.

$$\min_{\mathbf{H}} \|\mathbf{HX} - \widehat{\mathbf{X}}\|^2$$

is provided in this section. This separation leads directly to a formalization of an error metric.

From Equation (5),

$$\begin{aligned} & \|\mathbf{HX} - \widehat{\mathbf{X}}\|^2 = \\ = & \left\| \mathbf{R} \begin{pmatrix} R_0 & \dots & R_{n-1} \end{pmatrix} - \begin{pmatrix} \widehat{R}_0 & \dots & \widehat{R}_{n-1} \end{pmatrix} \right\|^2 + \textstyle\sum_{i=0}^{n-1} \|\mathbf{R}t_i + \mathbf{t} - \widehat{t}_i\|^2 \\ = & \sum_{i=0}^{n-1} \left\|\mathbf{R}R_i - \widehat{R}_i\right\|^2 + \sum_{i=0}^{n-1} \left\|\mathbf{R}t_i + \mathbf{t} - \widehat{t}_i\right\|^2. \end{aligned}$$

Therefore, we have a separation of the orientations from the positions. Moreover, once the $\mathbf{R}$ and $\mathbf{t}$ of the homogeneous matrix $\mathbf{H}$ are calculated from the procedure outlined in Section II, a means to find how well $\mathbf{R}$ and $\mathbf{t}$ fit the data can be constructed. Notice that for the orientation

$$\begin{aligned} \left\|\mathbf{R}R_i - \widehat{R}_i\right\|^2 &= \|\mathbf{R}R_i\|^2 - 2\text{tr}\left(\mathbf{R}R_i\widehat{R}_i^T\right) + \|\widehat{R}_i\|^2 \\ &= 6 - 2\text{tr}\left(\mathbf{R}R_i\widehat{R}_i^T\right) \\ &= 6 - 2(1 + 2\cos\theta) \\ &\leq 8. \end{aligned}$$

since $\|R\|^2 = 3$ and $\text{tr}(R) = 1 + 2\cos\theta$ for any rotation matrix R with eigenvalues $\{1, \cos\theta \pm i\sin\theta\}$. Therefore, if the angle θ between the column space of $\mathbf{R}R_i$ and $\widehat{R}_i$ is approximately equal to 0, then $6 - 2(1 + 2\cos\theta) \approx 6 - 2(3) = 0$, whereas if the angle $\theta \approx \pi$ then $6 - 2(1 + 2\cos\theta) \approx 6 - 2(-1) = 8$. Therefore, a metric or *percentage of accuracy* to evaluate the orientation for a given homogeneous matrix $\mathbf{H}$ (hence rotation $\mathbf{R}$ and translation $\mathbf{t}$) can be calculated as

$$0 \leq 1 - \frac{1}{8}\left\|\mathbf{R}R_i - \widehat{R}_i\right\|^2 \leq 1.$$

A metric for the positions can be calculated in a similar way. In this case, we are interested in norm

$$\|\mathbf{R}t_i + \mathbf{t} - \widehat{t}_i\|^2.$$

In other words, we want to see how close the vector $\mathbf{R}t_i + \mathbf{t}$ is to $\widehat{t}_i$ for a given rotation $\mathbf{R}$ and translation $\mathbf{t}$. In order to construct a metric or *percentage of accuracy* for this data, we consider the dot product of the normalized vectors, i.e.

$$0 \leq \left| \frac{\widehat{t}_i^T(\mathbf{R}t_i + \mathbf{t})}{\|\widehat{t}_i\|\|\mathbf{R}t_i + \mathbf{t}\|} \right| \leq 1.$$

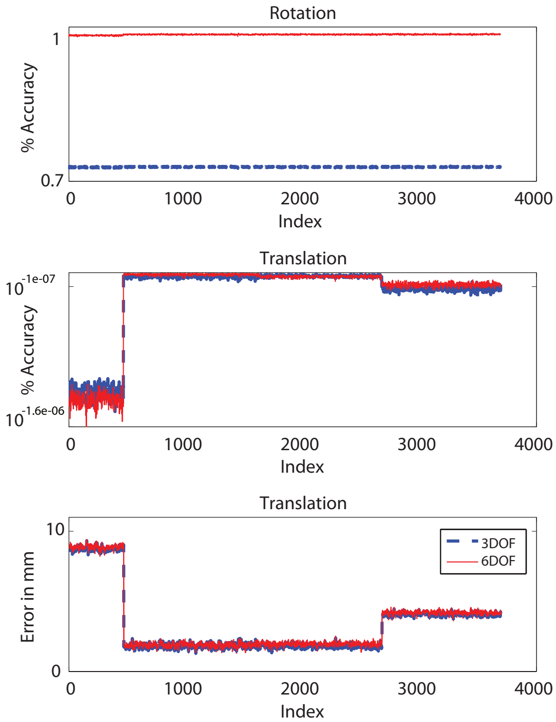

Fig. 1. 3DOF verses 6DOF on a linear dataset.

In other words, if the angle between the vectors is 0 we have 100% accuracy. A point of concern with this method is that the magnitude of the vectors are not taken into consideration. Therefore, this metric may exhibit 100% accuracy while the vectors are not exactly equal. Hence, one may want to compare the magnitude of

$$\|\mathbf{R}t_i + \mathbf{t} - \widehat{t_i}\|$$

with the magnitude of the positions t_i and $\widehat{t_i}$ to determine the accuracy of the algorithm. However, this metric does not have an upper-bound so it may be difficult to compare the results from different problem sets as is possible with the first metric presented.

IV. Experiments

A series of experiments were conducted at the Purdue Robot Vision Lab in April of 2008 which compared a 6DOF laser tracker (considered ground truth) with a real-time visual servoing system [7]. Problems arose with these experiments due to difficulties with hand-calibration of the 6DOF laser tracker with the real-time visual servoing system. Therefore, a means to mathematically calibrate the two systems was necessary.

The data streams from both systems were comprised of 6DOF data; however the streams were with respect to two different coordinate systems. Therefore, the method outlined in this paper was used in order to calibrate the systems. In other words, a homogeneous matrix $\mathbf{H}$ was produced to transform the real-time visual servoing system's data stream into the 6DOF laser tracker's coordinate system.

Experiments were conducted to compare the percentage of accuracy between the two systems using 3DOF (absolute orientation) and 6DOF (work presented here) using Matlab 7 on a Mac 2.16 GHz Intel Core 2 Duo machine. In Figure 1, the data stream consisted of data collected from a linear motion. As suggested in [1], infinite solutions exist for the solution of the 3DOF problem. In contrast, the 6DOF problem creates a unique solution with very high accuracy. The percentage of accuracy of the rotations with respect to 6DOF is nearly 100% while for 3DOF it hovers around 75%. With regards to the translations, the two procedures are nearly identical – both having very high accuracy. This is a result of the fact that both methods involve the term $\mathbf{T} = \mathbf{t} + \mathbf{R}t - \widehat{t}$ which can be arbitrarily set to 0 given any rotation $\mathbf{R}$. In addition, the translational error is at most 10 mm, which is a 2 digit reduction in size compared to the data which is in the 2000-3000 mm range. This reduction acknowledges a close fit of the translation results. It should be noted that the 3DOF solution presented in Figure 1 is the solution that Matlab 7 produced which is a result of slight noise in the collection of data.

V. Conclusion

In this paper, we constructed the best homogeneous matrix $\mathbf{H}$ to fit two streams of 6DOF data. In other words, we found the best rotation $\mathbf{R}$ and translation $\mathbf{t}$ that would transform the coordinate system of the first data stream to the second. We tested the algorithm on two data streams that modeled linear motion. We found that the algorithm constructed in this paper (6DOF) has a unique solution as opposed to the conventional absolution orientation (3DOF) model. In addition the *percentage of accuracy* is higher for the 6DOF algorithm when compared to the conventional 3DOF algorithm.

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